PROBABILITY

1 INTRODUCTION :

The theory of probability has been originated from the game of chance and gambling. In old days, gamblers used to gamble in a gambling house with a die to win the amount fixed among themselves. They were always desirous to get the prescribed number on the upper face of a die when it was thrown on a board. Shakuni of Mahabharat was perhaps one of them. People started to study the subject of probability from the middle of seventeenth century. The mathematicians Huygens, Pascal Fermat and Bernoulli contributed a lot to this branch of Mathematics. A.N. Kolmogorow proposed the set theoretic model to the theory of probability.

Probability gives us a measure of likelihood that something will happen. However probability can never predict the number of times that an occurrence actually happens. But being able to quantify the likely occurrence of an event is important because most of the decisions that affect our daily lives are based on likelihoods and not on absolute certainties.

2. DEFINITIONS :

(a) **Experiment** : An action or operation resulting in two or more well defined outcomes. e.g. tossing a coin, throwing a die, drawing a card from a pack of well shuffled playing cards etc.

(b) **Sample space** : A set S that consists of all possible outcomes of a random experiment is called a sample space and each outcome is called a sample point. Often, there will be more than one sample space that can describe outcomes of an experiment, but there is usually only one that will provide the most information.

e.g. in an experiment of "throwing a die", following sample spaces are possible :

(i) {even number, odd number}

(ii) {a number less than 3, a number equal to 3, a number greater than 3}

(iii) {1,2,3,4,5,6}

Here 3rd sample space is the one which provides most information.

If a sample space has a finite number of points it is called finite sample space and if it has an infinite number of points, it is called infinite sample space. e.g. (i) "in a toss of coin" either a head (H) or tail (T) comes up, therefore sample space of this experiment is S = {H,T} which is a finite sample space. (ii) "Selecting a number from the set of natural numbers", sample space of this experiment is S = {1,2,3,4,......} which is an infinite sample space.

(c) **Event** : An event is defined as an occurrence or situation, for example

(i) in a toss of a coin, it shows head,

(ii) scoring a six on the throw of a die,

(iii) winning the first prize in a raffle,

(iv) being dealt a hand of four cards which are all clubs.

In every case it is set of some or all possible outcomes of the experiment. Therefore event (A) is subset of sample space (S). If outcome of an experiment is an element of A we say that event A has occurred.

- An event consisting of a single point of S is called a simple or elementary event.
- ϕ is called impossible event and S (sample space) is called sure event.

Note : Probability of occurrence of an event A is denoted by P(A).

(d) **Compound Event** : If an event has more than one sample points it is called **Compound Event**. If A & B are two given events then $A \cap B$ is called compound event and is denoted by $A \cap B$ or AB or A & B.

(e) **Complement of an event :** The set of all outcomes which are in S but not in A is called the complement of the event A & denoted by $\bar{A}$, A^c, A' or 'not A'.

(f) **Mutually Exclusive Events :** Two events are said to be **Mutually Exclusive** (or disjoint or incompatible) if the occurrence of one precludes (rules out) the simultaneous occurrence of the other. If A & B are two mutually exclusive events then $P(A \cap B) = 0$.

Consider, for example, choosing numbers at random from the set {3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

If, Event A is the selection of a prime number,

Event B is the selection of an odd number,

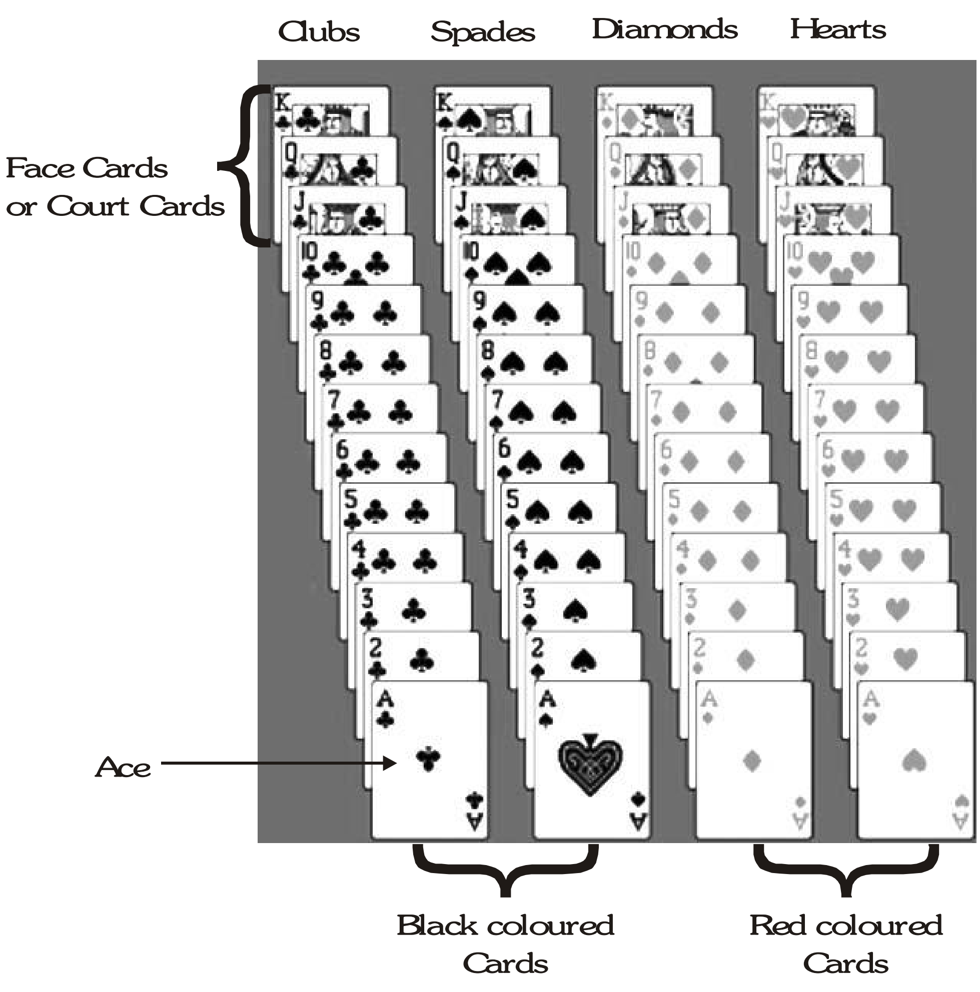
Event C is the selection of an even number,

then A and C are mutually exclusive as none of the numbers in this set is both prime and even. But A and B are not mutually exclusive as some numbers are both prime and odd (viz. 3, 5, 7, 11).

(g) **Equally Likely Events :** Events are said to be **Equally Likely** when each event is as likely to occur as any other event. Note that the term 'at random' or 'randomly' means that all possibilities are equally likely.

(h) **Exhaustive Events :** Events A,B,C........ N are said to be **Exhaustive Events** if no event outside this set can result as an outcome of an experiment. For example, if A & B are two events defined on a sample space S and A & B are exhaustive $\Rightarrow A \cup B = S \Rightarrow P(A \cup B) = 1$.

Note : Playing cards : A pack of playing cards consists of 52 cards of 4 suits, 13 in each, as shown in figure.



Comparative study of Equally likely, Mutually Exclusive and Exhaustive events :

	Experiment	Events	E/L	M/E	Exhaustive
1.	Throwing of a die	A: throwing an odd face {1, 3, 5} B : throwing a composite {4,6}	No	Yes	No
2.	A ball is drawn from an urn containing 2White, 3Red and 4Green balls	E_1 : getting a White ball E_2 : getting a Red ball E_3 : getting a Green ball	No	Yes	Yes
3.	Throwing a pair of dice	A : throwing a doublet {11, 22, 33, 44, 55, 66} B : throwing a total of 10 or more { 46, 64, 55, 56, 65, 66 }	Yes	No	No
4.	From a well shuffled pack of cards a card is drawn	E_1 : getting a heart E_2 : getting a spade E_3 : getting a diamond E_4 : getting a club	Yes	Yes	Yes
5.	From a well shuffled pack of cards a card is drawn	A = getting a heart B = getting a face card	No	No	No

Illustration 1 : A coin is tossed. If it shows head, we draw a ball from a bag consisting of 3 blue and 4 white balls; if it shows tail we throw a die. Describe the sample space of this experiment.

Solution : Let us denote blue balls by B_1, B_2, B_3 and the white balls by W_1, W_2, W_3, W_4. Then a sample space of the experiment is

S = {HB_1, HB_2, HB_3, HW_1, HW_2, HW_3, HW_4, T1, T2, T3, T4, T5, T6}.

Here HB_i means head on the coin and ball B_i is drawn, HW_i means head on the coin and ball W_i is drawn. Similarly, Ti means tail on the coin and the number i on the die.

Illustration 2 : Consider the experiment in which a coin is tossed repeatedly until a head comes up. Describe the sample space.

Solution : In the experiment head may come up on the first toss, or the 2nd toss, or the 3rd toss and so on. Hence, the desired sample space is S = {H, TH, TTH, TTTH, TTTTH,...}

Illustration 3 : A coin is tossed three times, consider the following events.

A : 'no head appears'

B : 'exactly one head appears'

C : 'at least two heads appear'

Do they form a set of mutually exclusive and exhaustive events ?

Solution : The sample space of the experiment is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Events A, B and C are given by

A = {TTT}

B = {HTT, THT, TTH}

C = {HHT, HTH, THH, HHH}

Now,

$A \cup B \cup C$ = {TTT, HTT,THT,TTH,HHT,HTH,THH,HHH} = S

Therefore A,B and C are exhaustive events. Also, $A \cap B = \phi$, $A \cap C = \phi$ and $B \cap C = \phi$. Therefore, the events are pair-wise disjoint, i.e., they are mutually exclusive. Hence, A,B and C form a set of mutually exclusive and exhaustive events.

Do yourself - 1 :

(i) Two balls are drawn from a bag containing 2 Red and 3 Black balls, write sample space of this experiment.

(ii) Out of 2 men and 3 women a team of two persons is to be formed such that there is exactly one man and one woman. Write the sample space of this experiment.

(iii) A coin in tossed and if head comes up, a die is thrown. But if tail comes up, the coin is tossed again. Write the sample space of this experiment.

(iv) In a toss of a die, consider following events :

A : An even number turns up.

B : A prime number turns up.

These events are -

(A) Equally likely events (B) Mutually exclusive events

(C) Exhaustive events (D) None of these

3. CLASSICAL DEFINITION OF PROBABILITY :

If n represents the total number of equally likely, mutually exclusive and exhaustive outcomes of an experiment and m of them are favourable to the happening of the event A, then the probability of happening of the event A is given by P(A) = m/n. There are (n–m) outcomes which are favorable to the event that A does not happen.

'The event A does not happen' is denoted by $\bar{A}$ (and is read as 'not A')

Thus $P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n}$

i.e. $P(\bar{A}) = 1 - P(A)$

Note :

(i) $0 \le P(A) \le 1$

(ii) $P(A) + P(\bar{A}) = 1$, Where $\bar{A}$ = Not A ; This relationship is most useful in the 'at least one' type of problems.

(iii) If x cases are favourable to A & y cases are favourable to $\bar{A}$ then $P(A) = \frac{x}{(x+y)}$ and $P(\bar{A}) = \frac{y}{(x+y)}$. We say that Odds In Favour Of A are x: y & Odds Against A are y : x

Note :

OTHER DEFINITIONS OF PROBABILITY :

(a) **Axiomatic probability :** Axiomatic approach is another way of describing probability of an event. In this approach some axioms or rules are depicted to assign probabilities.

Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval [0, 1] satisfying the following axioms :

(i) For any event E, $P(E) \ge 0$ (ii) $P(S) = 1$

(iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

It follows from (iii) that $P(E \cap F) = P(\phi) = 0$.

Let S be a sample space containing outcomes $\omega_1, \omega_2, \ldots, \omega_n$, i.e., $S = \{\omega_1, \omega_2, \ldots \omega_n\}$

It follows from the axiomatic definition of probability that :

(i) $0 \le P(\omega_i) \le 1$ for each $\omega_i \in S$

(ii) $P(\omega_1) + P(\omega_2) + \ldots + P(\omega_n) = 1$

(iii) For any event A, $P(A) = \Sigma P(\omega_i)$, $\omega_i \in A$.

(b) **Empirical probability :** The probability that you would hit the bull's-eye on a dartboard with one throw of a dart would depend on how much you had practised, how much natural talent for playing darts you had, how tired you were, how good a dart you were using etc. all of which are impossible to quantify. A method which can be adopted in the example given above is to throw the dart several times (each throw is a trial) and count the number of times you hit the bull's-eye (a success) and the number of times you miss (a failure). Then an empirical value of the probability that you hit the bull's-eye with any one throw is $\dfrac{\text{number of successes}}{\text{number of successes + number of failures}}$.

If the number of throws is small, this does not give a particular good estimate but for a large number of throws the result is more reliable.

When the probability of the occurrence of an event A cannot be worked out exactly, an empirical value can be found by adopting the approach described above, that is :

(i) making a large number of trials (i.e. set up an experiment in which the event may, or may not, occur and note the outcome),

(ii) counting the number of times the event does occur, i.e. the number of successes,

(iii) calculating the value of $\dfrac{\text{number of successes}}{\text{number of trials (i.e. successes + failures)}} = \dfrac{r}{n}$

The probability of event A occurring is defined as $P(A) = \lim\limits_{n\to\infty}\left(\dfrac{r}{n}\right)$

$n \to \infty$ means that the number of trials is large (but what should be taken as 'large' depends on the problem).

Illustration 4 : A coin is tossed successively three times. Find the probability of getting exactly one head or two heads.

Solution : Let S be the sample space and E be the event of getting exactly one head or exactly two heads, then

S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}.

and E = {HHT, HTH, THH, HTT, THT, TTH}

$\therefore$ n(E) = 6 and n(S) = 8.

Now required probability, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{6}{8} = \dfrac{3}{4}$.

Illustration 5 : Words are formed with the letters of the word PEACE. Find the probability that 2 E's come together.

Solution : Total number of words which can be formed with the letters P, E, A, C, E $= \dfrac{5!}{2!} = 60$

Number of words in which 2 E's come together $= 4! = 24$ $\therefore$ reqd. prob. $= \dfrac{24}{60} = \dfrac{2}{5}$ **Ans.**

Illustration 6 : A bag contains 5 red and 4 green balls. Four balls are drawn at random, then find the probability that two balls are of red and two balls are of green colour.

Solution : n(s) = the total number of ways of drawing 4 balls out of total 9 balls : $^9C_4$

A : Drawing 2 red and 2 green balls ; n(A) = $^5C_2$ $^4C_2$

$$\therefore \quad P(A) = \frac{n(A)}{n(s)} = \frac{^5C_2 \times {}^4C_2}{^9C_4} = \frac{\dfrac{5\times4\times4\times3}{2\times2}}{\dfrac{9\times8\times7\times6}{4\times3\times2}} = \frac{10}{21}$$ **Ans.**

Illustration 7 : If the letters of INTERMEDIATE are arranged, then the odds in favour of the event that no two 'E's occur together, are -

(A) $\dfrac{6}{5}$ (B) $\dfrac{5}{6}$ (C) $\dfrac{2}{9}$ (D) none of these

Solution : I → 2, N → 1, T → 2, E → 3, R → 1, M → 1, D → 1,A → 1 (3'E's, Rest 9 letters)

First arrange rest of the letters $= \dfrac{9!}{2!\ 2!}$,

Now 3'E's can be placed by ${}^{10}C_3$ ways, so favourable cases = $\dfrac{9!}{2!\ 2!} \times {}^{10}C_3 = 3 \times 10!$

Total cases = $\dfrac{12!}{2!\,2!\,3!} = \dfrac{11}{2} \times 10!$; Non-favourable cases = $\left(\dfrac{11}{2} - 3\right) \times 10! = \dfrac{5}{2} \times 10!$

Odds in favour of the event = $\dfrac{3}{5/2} = \dfrac{6}{5}$ **Ans. (A)**

Illustration 8 : From a group of 10 persons consisting of 5 lawyers, 3 doctors and 2 engineers, four persons are selected at random. The probability that the selection contains at least one of each category is-

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{3}$ (D) none of these

Solution : n(S) = ${}^{10}C_4$ = 210.

n(E) = ${}^{5}C_2\ \ {}^{3}C_1\ \ {}^{2}C_1 + {}^{5}C_1\ \ {}^{3}C_2\ \ {}^{2}C_1 + {}^{5}C_1\ \ {}^{3}C_1\ \ {}^{2}C_2 = 105$

∴ P(E) $= \dfrac{105}{210} = \dfrac{1}{2}$. **Ans. (A)**

Illustration 9 : If four cards are drawn at random from a pack of fifty-two playing cards, find the probability that at least one of them is an ace.

Solution : If A is a combination of four cards containing at least one ace (i.e. either one ace, or two aces, or three aces or four aces) then $\bar{A}$ is a combination of four cards containing no aces.

Now $P(\bar{A}) = \dfrac{\text{Number of combinations of four cards with no aces}}{\text{Total number of combinations of four cards}} = {}^{48}C_4 / {}^{52}C_4 = 0.72$

Using $P(A) + P(\bar{A}) = 1$ we have $P(A) = 1 - P(\bar{A}) = 1 - 0.72 = 0.28$

Illustration 10 : If n positive integers taken at random are multiplied together, show that the probability that the last digit of the product is 5 is $\dfrac{5^n - 4^n}{10^n}$ and that the probability of the last digit being 0 is $\dfrac{10^n - 8^n - 5^n + 4^n}{10^n}$.

Solution : Let n positive integers be $x_1, x_2, \ldots, x_n$. Let $a = x_1.x_2 \ldots x_n$.

Let S be the sample space, since the last digit in each of the numbers, $x_1, x_2, \ldots, x_n$ can be any one of the digits 0, 1, 2, 3,...,9 (total 10)

∴ $n(S) = 10^n$

Let E_1 and E_2 be the events when the last digit in a is 1, 3, 5, 7 or 9 and 1, 3, 7 or 9 respectively

∴ $n(E_1) = 5^n$ and $n(E_2) = 4^n$

and let E be the event that the last digit in a is 5.

$n(E) = n(E_1) - n(E_2) = 5^n - 4^n$

Hence required probability $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{5^n - 4^n}{10^n}$

Second part : Let E_3 and E_4 be the events when the last digit in a is 1, 2, 3, 4, 6, 7, 8 or 9 and 0 respectively.

Then $n(E_4) = n(S) - n(E_3) - n(E) = 10^n - 8^n - (5^n - 4^n) = 10^n - 8^n - 5^n + 4^n$

$\therefore$ Required probability $P(E_4) = \dfrac{n(E_4)}{n(S)} = \dfrac{10^n - 8^n - 5^n + 4^n}{10^n}$ **Ans.**

Illustration 11 : A bag contains n white and n red balls. Pairs of balls are drawn without replacement until the bag is empty. Show that the probability that each pair consists of one white and one red ball is $2^n/({}^{2n}C_n)$.

Solution : Let S be the sample space & E be the event that each of the n pairs of balls drawn consists of one white and one red ball.

$\therefore$ $n(S) = ({}^{2n}C_2)\,({}^{2n-2}C_2)\,({}^{2n-4}C_2)\ldots({}^{4}C_2)({}^{2}C_2)$

$$= \left\{\frac{(2n)(2n-1)}{1.2}\right\}\left\{\frac{(2n-2)(2n-3)}{1.2}\right\}\left\{\frac{(2n-4)(2n-5)}{1.2}\right\}\ldots\ldots\left\{\frac{4.3}{1.2}\right\}\left\{\frac{2.1}{1.2}\right\}$$

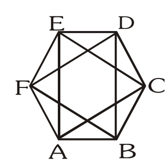
$$= \frac{1.2.3.4\ldots.(2n-1)2n}{2^n} = \frac{2n!}{2^n}$$

and $n(E) = ({}^{n}C_1.{}^{n}C_1)\,({}^{n-1}C_1.{}^{n-1}C_1)\,({}^{n-2}C_1.{}^{n-2}C_1)\ldots({}^{2}C_1.{}^{2}C_1)\,({}^{1}C_1.{}^{1}C_1)$

$= n^2.(n-1)^2.(n-2)^2\ldots.2^2.1^2 = [1.2.3\ldots..(n-1)n]^2 = (n!)^2$

$\therefore$ Required Probability, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{(n!)^2}{(2n)!/2^n} = \dfrac{2^n}{\frac{2n!}{(n!)^2}} = \dfrac{2^n}{{}^{2n}C_n}$ **Ans.**

Illustration 12 : Three vertices out of six vertices of a regular hexagon are chosen randomly. The probability of getting a equilateral triangle after joining three vertices is -



(A) 1/5 (B) 1/20

(C) 1/10 (D) 1/2

Solution : The total no. of cases = ${}^{6}C_3 = 20$

As shown in the figure only two triangles ACE and BDF are equilateral. So number of favourable cases is 2.

Hence the required probability = $\dfrac{2}{20} = \dfrac{1}{10}$ **Ans. (C)**

Do yourself - 2 :

(i) Find the probability of scoring a total of more than 7, when two dice are thrown.

(ii) A card is drawn randomly from a well shuffled pack of 52 playing cards and following events are defined:

A : The drawn card is a face card.

B : The drawn card is a spade.

Find odds in favour of A and odds against B.

(iii) Two natural numbers are selected at random, find the probability that their sum is divisible by 10.

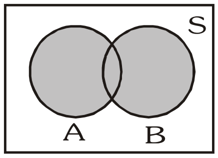
(iv) Five card are drawn successively from a pack of 52 cards with replacement. Find the probability that there is at least one Ace.

4. VENN DIAGRAMS :

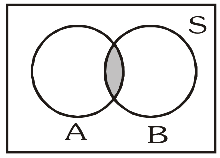
A diagram used to illustrate relationships between sets. Commonly, a rectangle represents the universal set and a circle within it represents a given set (all members of the given set are represented by points within the circle). A subset is represented by a circle within a circle and intersection is indicated by overlapping circles.

Let S is the sample space of an experiment and A, B are two events corresponding to it :

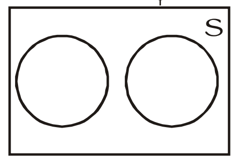
$A \cup B$



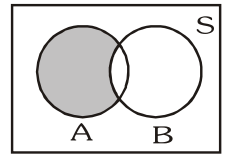
$A \cap B$



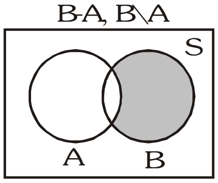
$A \cap B = \phi$



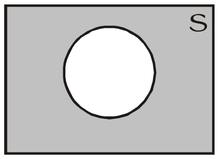
$A - B$ or $A \backslash B$ or $A \cap \overline{B}$



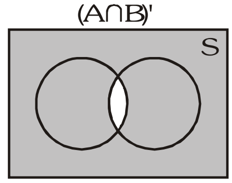
$B - A$, $B \backslash A$



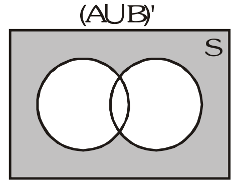
A', A^C or $\overline{A}$



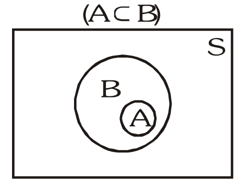
$(A \cap B)'$



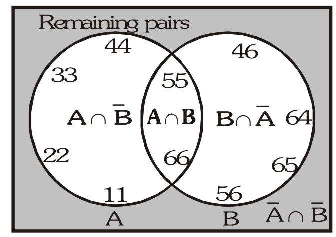
$(A \cup B)'$



$(A \subset B)$



Example : Let us now conduct an experiment of tossing a pair of dice. Two events defined on the experiment are



A : getting a doublet {11, 22, 33, 44, 55, 66}

B : getting total score of 10 or more {64, 46, 55, 56, 65, 66}

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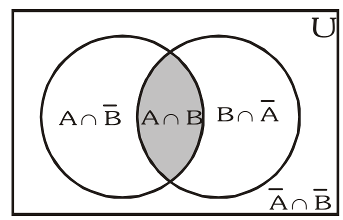
5. ADDITION THEOREM :



$A \cup B = A + B = A$ or B denotes occurrence of at least A or B.

For 2 events A & B :

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note :

(a)

$$\left.\begin{array}{l} P(A \cup B) \\ P(A + B) \\ P(A \text{ or } B) \\ P(\text{occurence of atleast A or B}) \end{array}\right] = \begin{array}{l} P(A) + P(B) - P(A \cap B) \text{ (This is known as generallised addition theorem)} \\ P(A) + P(B \cap \overline{A}) \\ P(B) + P(A \cap \overline{B}) \\ P(A \cap \overline{B}) + P(A \cap B) + P(B \cap \overline{A}) \\ 1 - P(A^C \cap B^C) \\ 1 - P(A \cup B)^C \end{array}$$

(b) $P(A \backslash B) = P(A - B) = P(A \cap B^C) = P(A) - P(A \cap B)$

(c) Opposite of "atleast A or B" is neither A nor B

i.e. $\overline{A + B} = 1 - (A \text{ or } B) = \overline{A} \cap \overline{B}$

Note that $P(A \cup B) + P(\overline{A} \cap \overline{B}) = 1$.

(d) If A & B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.

(e) For any two events A & B, P(exactly one of A, B occurs)

$= P\left(A \cap \overline{B}\right) + P\left(B \cap \overline{A}\right) = P(A) + P(B) - 2P(A \cap B)$

$= P\left(A \cup B\right) - P\left(A \cap B\right) = P\left(A^c \cup B^c\right) - P\left(A^c \cap B^c\right)$

(f) $\left(A - B\right) \cup \left(B - A\right) = \left(A \cup B\right) - \left(A \cap B\right)$

(g) **De Morgan's Law** : If A & B are two subsets of a universal set U, then

(i) $(A \cup B)^c = A^c \cap B^c$ & (ii) $(A \cap B)^c = A^c \cup B^c$

(h) $\left(A \cup B \cup C\right)^C = A^C \cap B^C \cap C^C$ & $\left(A \cap B \cap C\right)^C = A^C \cup B^C \cup C^C$

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ & $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Illustration ***13 :*** Given two events A and B. If odds against A are as 2 : 1 and those in favour of $A \cup B$ are as 3 : 1, then find the range of P(B).

Solution : Clearly $P(A) = 1/3$, $P(A \cup B) = 3/4$.

Now, $P(B) \leq P(A \cup B)$

$\Rightarrow \quad P(B) \leq 3/4$

Also, $P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$\Rightarrow \quad P(B) \geq P(A \cup B) - P(A) \quad (\because P(A \cap B) \geq 0)$

$\Rightarrow \quad P(B) \geq 3/4 - 1/3$

$\Rightarrow \quad P(B) \geq \frac{5}{12}$

$\Rightarrow \quad \frac{5}{12} \leq P(B) \leq \frac{3}{4}$ **Ans.**

Illustration ***14 :*** If A and B are two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(A^c) = \frac{2}{3}$. Then find -

(i) P(A) (ii) P(B) (iii) $P(A \cap B^c)$ (iv) $P(A^c \cap B)$

Solution : $P(A) = 1 - P(A^c) = 1 - \frac{2}{3} = \frac{1}{3}$

$P(B) = P(A \cup B) + P(A \cap B) - P(A) = \frac{3}{4} + \frac{1}{4} - \frac{1}{3} = \frac{2}{3}$

$P(A \cap B^c) = P(A) - P(A \cap B) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$P(A^c \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$ **Ans.**

Illustration ***15 :*** Three numbers are chosen at random without replacement from 1, 2, 3,.......,10. The probability that the minimum of the chosen numbers is 4 or their maximum is 8, is -

(A) $\frac{11}{40}$ (B) $\frac{3}{10}$ (C) $\frac{1}{40}$ (D) none of these

Solution : The probability of 4 being the minimum number $= \frac{^6C_2}{^{10}C_3}$

(because, after selecting 4 any two can be selected from 5, 6, 7, 8, 9, 10).

The probability of 8 being the maximum number $= \frac{^7C_2}{^{10}C_3}$.

The probability of 4 being the minimum number and 8 being the maximum number $= \frac{3}{^{10}C_3}$

$\therefore$ the required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{^6C_2}{^{10}C_3} + \frac{^7C_2}{^{10}C_3} - \frac{3}{^{10}C_3} = \frac{11}{40}$. **Ans. (A)**

Do yourself - 3 :

(i) Draw Venn diagram of (a) $(A^C \cap B^C) \cup (A \cap B)$ (b) $B^C \cup (A^C \cap B)$

(ii) If A and B are two mutually exclusive events, then-

(A) $P(A) \leq P(\bar{B})$ (B) $P(\bar{A} \cap \bar{B}) = P(\bar{A}) - P(B)$ (C) $P(\bar{A} \cup \bar{B}) = 0$ (D) $P(\bar{A} \cap B) = P(B)$

(iii) A bag contains 6 white, 5 black and 4 red balls. Find the probability of getting either a white or a black ball in a single draw.

(iv) In a class of 125 students, 70 passed in English, 55 in mathematics and 30 in both. Find the probability that a student selected at random from the class has passed in (a) at least one subject (b) only one subject.

6. CONDITIONAL PROBABILITY AND MULTIPLICATION THEOREM :

Let A and B be two events such that P (A) >0. Then P(B|A) denote the conditional probability of B given that A has occurred. Since A is known to have occurred, it becomes the new sample space replacing the original S. From this we led to the definition.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

which is called conditional probability of B given A

$\Rightarrow$ $P(A \cap B) = P(A)$ P (B|A) which is called compound probability or multiplication theorem. It says the probability that both A and B occur is equal to the probability that A occur times the probability that B occurs given that A has occurred.

Note : For any three events A_1, A_2, A_3 we have $P(A_1 \cap A_2 \cap A_3)$ = $P(A_1)$ $P(A_2|A_1)$ $P(A_3 | (A_1 \cap A_2))$

Illustration 16 : Two dice are thrown. Find the probability that the numbers appeared have a sum of 8 if it is known that the second die always exhibits 4

Solution : Let A be the event of occurrence of 4 always on the second die

$= \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)\}$; $\therefore\ n(A) = 6$

and B be the event of occurrence of such numbers on both dice whose sum is 8 = {(6,2), (5,3), (4,4), (3,5), (2,6)}.

Thus, $A \cap B = A \cap \{(4,4)\} = \{(4,4)\}$

$\therefore\ n(A \cap B) = 1$

$$\therefore\ P(B/A) = \frac{n(A \cap B)}{n(A)} = \frac{1}{6} \text{ or } \frac{P(A \cap B)}{P(A)} = \frac{1/36}{6/36} = \frac{1}{6}$$

Illustration 17 : A bag contains 3 red, 6 white and 7 blue balls. Two balls are drawn one by one. What is the probability that first ball is white and second ball is blue when first drawn ball is not replaced in the bag?

Solution : Let A be the event of drawing first ball white and B be the event of drawing second ball blue. Here A and B are dependent events.

$$P(A) = \frac{6}{16},\ P(B | A) = \frac{7}{15}$$

$$P(AB) = P(A).P(B | A) = \frac{6}{16} \times \frac{7}{15} = \frac{7}{40}$$

Illustration 18 : A bag contains 4 red and 4 blue balls. Four balls are drawn one by one from the bag, then find the probability that the drawn balls are in alternate colour.

Solution : E_1 : Event that first drawn ball is red, second is blue and so on.

E_2 : Event that first drawn ball is blue, second is red and so on.

$$\therefore\ P(E_1) = \frac{4}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \text{ and } P(E_2) = \frac{4}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5}$$

$$P(E) = P(E_1) + P(E_2) = 2 \times \frac{4}{8}.\frac{4}{7}.\frac{3}{6}.\frac{3}{5} = \frac{6}{35}$$ **Ans.**

Illustration 19 : If two events A and B are such that $P(\bar{A}) = 0.3$, P(B) = 0.4 and $P(A\bar{B}) = 0.5$ then $P(B\,|\,(A \cup \bar{B}))$ equals -

(A) 1/2 (B) 1/3 (C) 1/4 (D) 1/5

Solution : We have $P(B\,|\,(A \cup \bar{B})) = \dfrac{P[B \cap (A \cup \bar{B})]}{P(A \cup \bar{B})} = \dfrac{P[(B \cap A) \cup (B \cap \bar{B})]}{P(A) + P(\bar{B}) - P(A \cap \bar{B})}$

$$= \frac{P(AB)}{P(A) + P(\bar{B}) - P(A\bar{B})} = \frac{P(A) - P(A\bar{B})}{P(A) + P(\bar{B}) - P(A\bar{B})} = \frac{0.7 - 0.5}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$$

Ans. (C)

Do yourself - 4 :

(i) A bag contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the bag and kept aside. From the remaining balls another ball is drawn and kept aside the first. This process is repeated till all the balls are drawn. Then probability that the balls drawn are in sequence of 2 black, 4 white and 3 red is-

(A) $\dfrac{1}{1260}$ (B) $\dfrac{1}{7560}$ (C) $\dfrac{1}{210}$ (D) None of these

(ii) Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that the drawn cards are face cards of same suit ?

7. INDEPENDENT EVENTS :

Two events A & B are said to be independent if occurrence or non occurrence of one does not affect the probability of the occurrence or non occurrence of other.

(a) If the occurrence of one event affects the probability of the occurrence of the other event then the events are said to be **Dependent** or **Contingent.** For two independent events A and B :
$P(A \cap B) = P(A)\,.\,P(B)$. Often this is taken as the definition of independent events.

(b) Three events A, B & C are independent if & only if all the following conditions hold ;
$P(A \cap B) = P(A)\,.\,P(B)$; $P(B \cap C) = P(B)\,.\,P(C)$
$P(C \cap A) = P(C)\,.\,P(A)$ & $P(A \cap B \cap C) = P(A)\,.\,P(B)\,.\,P(C)$
i.e. they must be pairwise as well as mutually independent.
Similarly for n events $A_1, A_2, A_3, \ldots\ldots A_n$ to be independent, the number of these conditions is equal to ${}^nC_2 + {}^nC_3 + \ldots\ldots + {}^nC_n = 2^n - n - 1$.
Note : Independent events are not in general mutually exclusive & vice versa.
Mutually exclusiveness can be used when the events are taken from the same experiment & independence can be used when the events are taken from different experiments.

Illustration 20 : The probability that an anti aircraft gun can hit an enemy plane at the first, second and third shot are 0.6, 0.7 and 0.1 respectively. The probability that the gun hits the plane is

(A) 0.108 (B) 0.892 (C) 0.14 (D) none of these

Solution : Let the events of hitting the enemy plane at the first, second and third shot are respectively A, B and C. Then as given $P(A) = 0.6, P(B) = 0.7, P(C) = 0.1$

Since $P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$ and events A,B,C are independent

$\Rightarrow P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A})P(\bar{B})P(\bar{C})$

Required probability = $P(A \cup B \cup C) = 1 - P(\bar{A})P(\bar{B})P(\bar{C})$

$= 1 - (1 - 0.6)(1 - 0.7)(1 - 0.1) = 1 - (0.4)(0.3)(0.9) = 1 - 0.108 = 0.892$ **Ans. (B)**

Illustration 21 : A lot contains 50 defective and 50 non-defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as : **[IIT 1992]**

A = {The first bulb is defective}

B = {The second bulb is non-defective}

C = {The two bulbs are both defective or both non-defective}

Determine whether

(i) A, B, C are pairwise independent, (ii) A, B, C are independent.

Solution : We have $P(A) = \frac{50}{100}.1 = \frac{1}{2}$; $P(B) = 1.\frac{50}{100} = \frac{1}{2}$; $P(C) = \frac{50}{100}.\frac{50}{100} + \frac{50}{100}.\frac{50}{100} = \frac{1}{2}$

$A \cap B$ is the event that first bulb is defective and second is non-defective.

$\therefore \quad P(A \cap B) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$

$A \cap C$ is the event that both bulbs are defective.

$\therefore \quad P(A \cap C) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$

Similarly $P(B \cap C) = \frac{1}{4}$

Thus we have $P(A \cap B) = P(A) . P(B)$; $P(A \cap C) = P(A) . P(C)$; $P(B \cap C) = P(B) . P(C)$

$\therefore$ A, B and C are pairwise independent.

There is no element in $A \cap B \cap C$

$\therefore \quad P(A \cap B \cap C) = 0$

$\therefore \quad P(A \cap B \cap C) \neq P(A) . P(B) . P(C)$

Hence A, B and C are not mutually independent.

Do yourself - 5 :

(i) For two independent events A and B, the probability that both A & B occur is 1/8 and the probability that neither of them occur is 3/8. The probability of occurrence of A may be -

(A) 1/2 (B) 1/4 (C) 1/8 (D) 3/4

(ii) A die marked with numbers 1,2,2,3,3,3 is rolled three times. Find the probability of occurrence of 1,2 and 3 respectively.

(iii) If A and B are two independent events, then prove that the following events are also independent -

(a) A' and B (b) A and B' (c) A' and B'

8. TOTAL PROBABILITY THEOREM :

Let an event A occurs with one of the n mutually exclusive & exhaustive events $B_1, B_2, B_3, \ldots\ldots\ldots B_n$ then $A = AB_1 + AB_2 + AB_3 + \ldots\ldots. + AB_n$

$$P(A) = P(AB_1) + P(AB_2) + \ldots\ldots. + P(AB_n) = \sum_{i=1}^{n} P(AB_i)$$

$$P(A) = P(B_1)\,P(A|B_1) + P(B_2)\,P(A|B_2) + \ldots\ldots\ldots\ldots + P(B_n)\,P(A|B_n) = \Sigma P(B_i)\,P(A|B_i)$$

Illustration 22 : A purse contains 4 copper and 3 silver coins and another purse contains 6 copper and 2 silver coins. One coin is drawn from any one of these two purses. The probability that it is a copper coin is -

(A) $\frac{4}{7}$ (B) $\frac{3}{4}$ (C) $\frac{2}{7}$ (D) $\frac{37}{56}$

Solution : Let A ≡ event of selecting first purse

B ≡ event of selecting second purse

C ≡ event of drawing a copper coin

Then given event has two disjoint cases: AC and BC

$$\therefore P(C) = P(AC + BC) = P(AC) + P(BC) = P(A)P(C \mid A) + P(B)P(C \mid B)$$

$$= \frac{1}{2}.\frac{4}{7} + \frac{1}{2}.\frac{6}{8} = \frac{37}{56}$$

Ans. (D)

Illustration 23 : Three groups A, B, C are contesting for positions on the Board of Directors of a Company. The probabilities of their winning are 0.5, 0.3, 0.2 respectively. If the group A wins, the probability of introducing a new product is 0.7 and the corresponding probabilities for group B and C are 0.6 and 0.5 respectively. Find the probability that the new product will be introduced.

Solution : Given P(A) = 0.5, P(B) = 0.3 and P(C) = 0.2

$\therefore$ P(A) + P(B) + P(C) =1

then events A, B, C are exhaustive.

If P(E) = Probability of introducing a new product, then as given

P(E|A) = 0.7, P(E|B) =0.6 and P(E|C) = 0.5

$\therefore$ P(E) = P(A). P(E|A) + P(B). P(E|B) + P(C). P(E|C)

= 0.5   0.7 + 0.3   0.6 + 0.2   0.5 = 0.35 + 0.18 + 0.10 = 0.63

Illustration 24 : A pair of dice is rolled together till a sum of either 5 or 7 is obtained. Find the probability that 5 comes before 7.

Solution : Let E_1 = the event of getting 5 in a roll of two dice = {(1, 4), (2, 3), (3, 2), (4, 1)}

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{6 \times 6} = \frac{1}{9}$$

Let E_2 = the event of getting either 5 or 7

= {(1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{6 \times 6} = \frac{5}{18}$$

$\therefore$ the probability of getting neither 5 nor 7 $= P(\bar{E}_2) = 1 - P(E_2) = 1 - \frac{5}{18} = \frac{13}{18}$

The event of getting 5 before 7 $= E_1 \cup (\bar{E}_2 E_1) \cup (\bar{E}_2 \bar{E}_2 E_1) \cup \ldots\ldots \text{ to } \infty$

$\therefore$ the probability of getting 5 before 7

$$= P(E_1) + P(\bar{E}_2 E_1) + P(\bar{E}_2 \bar{E}_2 E_1) + \ldots. \text{ to } \infty = P(E_1) + P(\bar{E}_2)P(E_1) + P(\bar{E}_2)P(\bar{E}_2)P(E_1) + \ldots. \text{ to } \infty$$

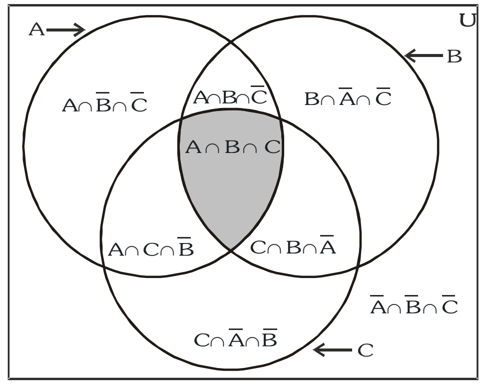
$$= \frac{1}{9} + \frac{13}{18}.\frac{1}{9} + \frac{13}{18}.\frac{13}{18}.\frac{1}{9} + \ldots\ldots \text{ to } \infty = \frac{1}{9}\left[1 + \frac{13}{18} + \left(\frac{13}{18}\right)^2 + \ldots\ldots \text{ to } \infty\right] = \frac{1}{9}.\frac{1}{1 - \frac{13}{18}} = \frac{1}{9}.\frac{18}{5} = \frac{2}{5}$$

Do yourself - 6 :

(i) An urn contains 6 white & 4 black balls. A die is rolled and the number of balls equal to the number obtained on the die are drawn from the urn. Find the probability that the balls drawn are all black.

(ii) There are n bags such that i^{th} bag $(1 \le i \le n)$ contains i black and 2 white balls. Two balls are drawn from a randomly selected bag out of given n bags. Find the probability that the both drawn balls are white.

9. PROBABILITY OF THREE EVENTS :



For any three events A,B and C we have

(a) $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \cap B)$

$- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(b) P(at least two of A,B,C occur)

$= P(B \cap C) + P(C \cap A) + P(A \cap B) - 2P(A \cap B \cap C)$

(c) P(exactly two of A,B,C occur)

$= P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$

(d) P(exactly one of A,B,C occurs)

$= P(A)+ P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

Note : If three events A, B and C are pair wise mutually exclusive then they must be mutually exclusive. i.e $P(A \cap B) = P(B \cap C) = P(C \cap A) = 0 \Rightarrow P(A \cap B \cap C) = 0$. However the converse of this is not true.

Illustration 25 : Let A,B,C be three events. If the probability of occurring exactly one event out of A and B is 1 – a, out of B and C is 1 – 2a, out of C and A is 1 – a and that of occurring three events simultaneously is a^2, then prove that the probability that at least one out of A,B,C will occur is greater than 1/2.

Solution : $P(A) + P(B) - 2P(A \cap B) = 1 - a$ .....(1)

and $P(B) + P(C) - 2P(B \cap C) = 1 - 2a$ .....(2)

and $P(C) + P(A) - 2P(C \cap A) = 1 - a$ .....(3)

and $P(A \cap B \cap C) = a^2$ .....(4)

$\because P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$$= \frac{1}{2}\{P(A) + P(B) - 2P(A \cap B) + P(B) + P(C) - 2P(B \cap C) + P(C) + P(A) - 2P(C \cap A)\} + P(A \cap B \cap C)$$

$$= \frac{1}{2}\{1 - a + 1 - 2a + 1 - a\} + a^2 \qquad \{\text{from (1), (2), (3) \& (4)}\}$$

$$= \frac{3}{2} - 2a + a^2 = (a - 1)^2 + \frac{1}{2} > \frac{1}{2}$$ **Ans.**

Do yourself - 7 :

(i) In a class, there are 100 students out of which 45 study mathematics, 48 study physics, 40 study chemistry, 12 study both mathematics & physics, 11 study both physics & chemistry, 15 study both mathematics & chemistry and 5 study all three subjects. A student is selected at random, then find the probability that the selected student studies

(a) only one subject (b) neither physics nor chemistry

10. BINOMIAL PROBABILITY DISTRIBUTION :

Suppose that we have an experiment such as tossing a coin or die repeatedly or choosing a marble from an urn repeatedly. Each toss or selection is called a trial. In any single trial there will be a probability associated with a particular event such as head on the coin, 4 on the die, or selection of a red marble. In some cases this probability will not change from one trial to the next (as in tossing a coin or die). Such trials are then said to be independent and are often called Bernoulli trials after James Bernoulli who investigated them at the end of the seventeenth century.

Let p be the probability that an event will happen in any single Bernoulli trial (called the probability of success). Then $q = 1 - p$ is the probability that the event will fail to happen in any single trial (called the probability of failure). The probability that the event will happen exactly x times in n trials (i.e., x successes and n – x failures will occur) is given by the probability function.

$$f(x) = P(X = x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!(n-x)!} p^x q^{n-x} \qquad \text{......... (i)}$$

where the random variable X denotes the number of successes in n trials and x = 0, 1,........., n.

Example : The probability of getting exactly 2 heads in 6 tosses of a fair coin is

$$P(X=2) = \binom{6}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{6-2} = \frac{6!}{2!4!}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{6-2} = \frac{15}{64}$$

The discrete probability function (i) is often called the binomial distribution since for x = 0, 1, 2,...,n, it corresponds to successive terms in the binomial expansion

$$(q+p)^n = q^n + \binom{n}{1} q^{n-1}p + \binom{n}{2} q^{n-2}p^2 + ... + p^n = \sum_{x=0}^{n} \binom{n}{x} p^x q^{n-x}$$

The special case of a binomial distribution with n = 1 is also called the Bernoulli distribution.

Illustration 26 : If a fair coin is tossed 10 times, find the probability of getting

(i) exactly six heads (ii) atleast six heads (iii) atmost six heads

Solution : The repeated tosses of a coin are Bernoulli trials. Let X denotes the number of heads in an experiment of 10 trials.

Clearly, X has the binomial distribution with n = 10 and $p = \frac{1}{2}$

Therefore $P(X = x) = {}^nC_x q^{n-x} p^x,\ x = 0, 1, 2, ..., n$

Here $n = 10,\ p = \frac{1}{2},\ q = 1 - p = \frac{1}{2}$

Therefore $P(X = x) = {}^{10}C_x \left(\frac{1}{2}\right)^{10-x}\left(\frac{1}{2}\right)^x = {}^{10}C_x \left(\frac{1}{2}\right)^{10}$

Now (i) $P(X = 6) = {}^{10}C_6 \left(\frac{1}{2}\right)^{10} = \frac{10!}{6!\times 4!2^{10}} = \frac{105}{512}$

(ii) P(atleast six heads) = $P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$

$$= {}^{10}C_6\left(\frac{1}{2}\right)^{10} + {}^{10}C_7\left(\frac{1}{2}\right)^{10} + {}^{10}C_8\left(\frac{1}{2}\right)^{10} + {}^{10}C_9\left(\frac{1}{2}\right)^{10} + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$$

$$= \left[\left(\frac{10!}{6!\times 4!}\right) + \left(\frac{10!}{7!\times 3!}\right) + \left(\frac{10!}{8!\times 2!}\right) + \left(\frac{10!}{9!\times 1!}\right) + \left(\frac{10!}{10!}\right)\right]\frac{1}{2^{10}} = \frac{193}{512}$$

(iii) P(at most six heads) = $P(X \leq 6)$

$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$

$$= \left(\frac{1}{2}\right)^{10} + {}^{10}C_1\left(\frac{1}{2}\right)^{10} + {}^{10}C_2\left(\frac{1}{2}\right)^{10} + {}^{10}C_3\left(\frac{1}{2}\right)^{10} + {}^{10}C_4\left(\frac{1}{2}\right)^{10} + {}^{10}C_5\left(\frac{1}{2}\right)^{10} + {}^{10}C_6\left(\frac{1}{2}\right)^{10}$$

$$= \frac{848}{1024} = \frac{53}{64}$$

Illustration 27 : India and Pakistan play a 5 match test series of hockey, the probability that India wins at least three matches is -

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{5}{16}$

Solution : India win atleast three matches $= {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_4\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^5(16) = \frac{1}{2}$ **Ans. (A)**

Illustration 28 : A coin is tossed 7 times. Each time a man calls head. The probability that he wins the toss on more than three occasions is -

(A) $\frac{1}{4}$ (B) $\frac{5}{8}$ (C) $\frac{1}{2}$ (D) none of these

Solution : The man has to win at least 4 times.

$\therefore$ the required probability

$$= {}^7C_4\left(\frac{1}{2}\right)^4\cdot\left(\frac{1}{2}\right)^3 + {}^7C_5\cdot\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2 + {}^7C_6\left(\frac{1}{2}\right)^6\cdot\frac{1}{2} + {}^7C_7\left(\frac{1}{2}\right)^7$$

$$= ({}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7)\cdot\frac{1}{2^7} = \frac{64}{2^7} = \frac{1}{2}.$$ **Ans. (C)**

Illustration 29 : A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

Solution : Since the man is one step away from starting point mean that either

(i) man has taken 6 steps forward and 5 steps backward.

(ii) man has taken 5 steps forward and 6 steps backward.

Taking, movement 1 step forward as success and 1 step backward as failure.

$\therefore$ p = Probability of success = 0.4

and q = Probability of failure = 0.6

$\therefore$ Required Probability = P {X = 6 or X = 5} = P (X = 6) + P(X = 5) $= {}^{11}C_6p^6q^5 + {}^{11}C_5p^5q^6$

$$= {}^{11}C_5(p^6q^5 + p^5q^6) = \frac{11.10.9.8.7}{1.2.3.4.5}\{(0.4)^6(0.6)^5 + (0.4)^5(0.6)^6\}$$

$$= \frac{11.10.9.8.7}{1.2.3.4.5}(0.24)^5$$

Hence the required probability = 0.37

Do yourself - 8 :

(i) An experiment succeeds twice as often as it fails. Find the probability that in next 6 trials, there will be more than 3 successes.

(ii) Find the probability of getting 4 exactly thrice in 7 throws of a die.

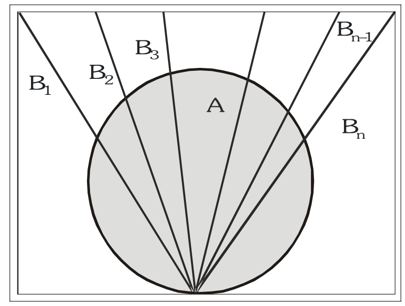
11. BAYE'S THEOREM :

If an event A can occur only with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots B_n$ & the probabilities $P(A/B_1), P(A/B_2)\ldots\ldots P(A/B_n)$ are known then,

$$P(B_i/A) = \frac{P(B_i).P(A/B_i)}{\sum_{i=1}^{n} P(B_i).P(A/B_i)}$$

Explanation :

A ≡ event what we have ; B_i ≡ event what we want & remaining are alternative events.



Now, $P(AB_i) = P(A).\ P(B_i/A) = P(B_i).\ P(A/B_i)$

$$P\left(B_i / A\right) = \frac{P(B_i)\ .\ P\left(A / B_i\right)}{P(A)} = \frac{P(B_i)\ .\ P\left(A / B_i\right)}{\sum_{i=1}^{n} P(AB_i)}$$

$$P\left(B_i / A\right) = \frac{P(B_i)\ .\ P\left(A / B_i\right)}{\sum_{i=1}^{n} P(B_i)\ .\ P\left(A / B_i\right)}$$

Illustration 30 : Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ?

Solution : Let E_1, E_2 and E_3 be the events that boxes I, II and III are chosen, respectively.

Then $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Also, let A be the event that 'the coin drawn is of gold'

Then $P(A|E_1)$ = P(a gold coin from box I) = $\frac{2}{2} = 1$

$P(A|E_2)$ = P(a gold coin from box II) = 0

$P(A|E_3)$ = P(a gold coin from box III) = $\frac{1}{2}$

Now, the probability that the other coin in the box is of gold

= the probability that gold coin is drawn from the box I.

= $P(E_1|A)$

By Baye's theorem, we know that

$$P(E_1 \mid A) = \frac{P(E_1)P(A \mid E_1)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2) + P(E_3)P(A \mid E_3)} = \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times 0 + \frac{1}{3} \times \frac{1}{2}} = \frac{2}{3}$$

Illustration 31 : In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B ?

Solution : Let events B_1, B_2, B_3 be the following :

B_1 : the bolt is manufactured by machine A

B_2 : the bolt is manufactured by machine B

B_3 : the bolt is manufactured by machine C

Clearly, B_1, B_2, B_3 are mutually exclusive and exhaustive events and hence, they represent a partition of the sample space.

Let the event E be ' the bolt is defective'.

The event E occurs with B_1 or with B_2 or with B_3.

Given that $P(B_1) = 25\% = 0.25$, $P(B_2) = 0.35$ and $P(B_3) = 0.40$

Again $P(E|B_1)$ = Probability that the bolt drawn is defective given that it is manufactured by machine A = 5% = 0.05.

Similarly, $P(E|B_2) = 0.04$, $P(E|B_3) = 0.02$.

Hence, by Baye's Theorem, we have

$$P(B_2 \mid E) = \frac{P(B_2)P(E \mid B_2)}{P(B_1)P(E \mid B_1) + P(B_2)P(E \mid B_2) + P(B_3)P(E \mid B_3)}$$

$$= \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02} = \frac{0.0140}{0.0345} = \frac{28}{69}$$

Illustration 32 : A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from the bag B.

Solution : Let E_1 = The event of ball being drawn from bag A

E_2 = The event of ball being drawn from bag B.

E = The event of ball being red.

Since, both the bags are equally likely to be selected, therefore

$$P(E_1) = P(E_2) = \frac{1}{2} \text{ and } P(E \mid E_1) = \frac{3}{5} \text{ and } P(E \mid E_2) = \frac{5}{9}$$

$$\therefore \quad \text{Required probability } P(E_2 \mid E) = \frac{P(E_2)P(E \mid E_2)}{P(E_1).P(E \mid E_1) + P(E_2)P(E \mid E_2)} = \frac{\frac{1}{2} \times \frac{5}{9}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{5}{9}} = \frac{25}{52}$$

Illustration 33 : In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that he correctly answered it.

[IIT 1991]

Solution : Let A_1 be the event that the examinee guesses that answer; A_2 the event that he copies the answer and A_3 the event that he knows the answer. Also let A be the event that he answers correctly. Then as given, we have

$P(A_1) = \frac{1}{3}$, $P(A_2) = \frac{1}{6}$, $P(A_3) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$

[We have assumed here that the events A_1, A_2 and A_3 are mutually exclusive and totally exhaustive.]

Now $P(A/A_1) = \frac{1}{4}$, $P(A / A_2) = \frac{1}{8}$ (as given)

Again it is reasonable to take the probability of answering correctly given that he knows the answer as 1, that is, $P(A/A_3) = 1$

We have to find $P(A_3/A)$.

By Baye's theorem, we have $P(A_3/A) = \frac{P(A_3)P(A / A_3)}{P(A_1)P(A / A_1) + P(A_2)P(A / A_2) + (A_3)P(A / A_3)}$

$$= \frac{(1/2).1}{(1/3)(1/4) + (1/6)(1/8) + (1/2).1} = \frac{24}{29}$$

Do yourself - 9 :

(i) A pack of cards was found to contain only 51 cards. If first 13 cards, which are examined, are all red, then find the probability that the missing cards is black.

(ii) A man has 3 coins A, B & C. A is fair coin. B is biased such that the probability of occurring head on it is 2/3. C is also biased with the probability of occurring head as 1/3. If one coin is selected and tossed three times, giving two heads and one tail, find the probability that the chosen coin was A.

12. PROBABILITY THROUGH STATISTICAL (STOCHATIC) TREE DIAGRAM :

These tree diagrams are generally drawn by economist and give a simple approach to solve a problem.

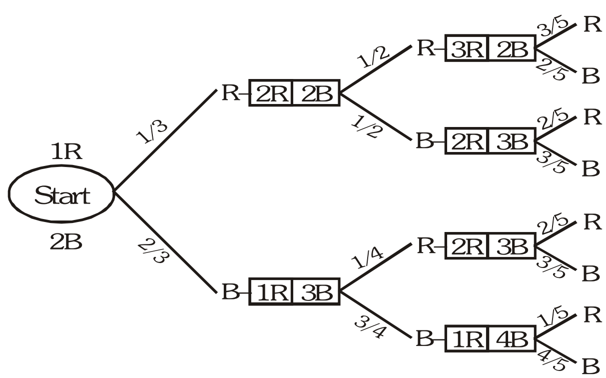
Illustration 34 : A bag initially contains 1 red ball and 2 blue balls. A trial consists of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. Given that three trials are made, draw a tree diagram illustrating the various probabilities. Hence, or otherwise, find the probability that

(a) atleast one blue ball is drawn

(b) exactly one blue ball is drawn

(c) Given that all three balls drawn are of the same colour find the probability that they are all red.

Solution :



Calculations :

$$P(A) = 1 - P(RRR) = 1 - \frac{1}{3}.\frac{1}{2}.\frac{3}{5} = 1 - \frac{1}{10} = \frac{9}{10}$$

$$P(\text{exactly one Blue}) = \frac{1}{3}.\frac{1}{2}.\frac{2}{5} + \frac{1}{3}.\frac{1}{2}.\frac{2}{5} + \frac{2}{3}.\frac{1}{4}.\frac{2}{5} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{3}{15} = \frac{1}{5}$$

$$P(C) = P\left(\frac{RRR}{(RRR \cup BBB)}\right) = \frac{P(RRR)}{P(RRR) + P(BBB)} = \frac{\frac{1}{3}.\frac{1}{2}.\frac{3}{5}}{\frac{1}{3}.\frac{1}{2}.\frac{3}{5} + \frac{2}{3}.\frac{3}{4}.\frac{4}{5}} = \frac{\frac{1}{10}}{\frac{1}{10} + \frac{4}{10}} = \frac{1}{5}$$

Do yourself - 10 :

(i) Three coins are tossed. Two of them are fair and one is biased so that a head is three times as likely as a tail. Find the probability of getting two heads and a tail.

(ii) In a multiple choice test of three questions there are five alternative answers given to the first two questions each and four alternative answers given to the last question. If a candidate guesses answers at random, what is the probability that he will get-

(a) Exactly one right ? (b) At least one right ?

13. COINCIDENCE TESTIMONY :

If p_1 and p_2 are the probabilities of speaking the truth of two independent witnesses A and B then

$$P \text{ (their combined statement is true)} = \frac{p_1 p_2}{p_1 p_2 + (1-p_1)(1-p_2)}.$$

In this case it has been assumed that we have no knowledge of the event except the statement made by A and B. However if p is the probability of the happening of the event before their statement then

$$P \text{ (their combined statement is true)} = \frac{p\, p_1 p_2}{p\, p_1 p_2 + (1-p)(1-p_1)(1-p_2)}.$$

Here it has been assumed that the statement given by all the independent witnesses can be given in two ways only, so that if all the witnesses tell falsehoods they agree in telling the same falsehood.

If this is not the case and c is the chance of their coincidence testimony then the

probability that the statement is true = $p\ p_1 p_2$

probability that the statement is false = $(1-p).c\ (1-p_1)(1-p_2)$

However chance of coincidence testimony is taken only if the joint statement is not contradicted by any witness.

Illustration 35 : A speaks truth in 75% cases and B in 80% cases. What is the probability that they contradict each other in stating the same fact?

(A) 7/20 (B) 13/20 (C) 3/20 (D) 1/5

Solution : There are two mutually exclusive cases in which they contradict each other i.e. $\bar{A}B$ and $A\bar{B}$.

Hence required probability = $P(A\bar{B} + \bar{A}B) = P(A\bar{B}) + P(\bar{A}B)$

$= P(A)P(\bar{B}) + P(\bar{A})P(B) = \frac{3}{4}\cdot\frac{1}{5} + \frac{1}{4}\cdot\frac{4}{5} = \frac{7}{20}$ **Ans. (A)**

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14. PROBABILITY DISTRIBUTION (Not in JEE) :

(a) A Probability Distribution spells out how a total probability of 1 is distributed over several values of a random variable.

(b) Mean of any probability distribution of a random variable is given by :

$$\mu = \frac{\sum p_i x_i}{\sum p_i} = \sum p_i x_i \quad (\text{ Since } \Sigma\, p_i = 1)$$

(c) Variance of a random variable is given by, $\sigma = \Sigma (x_i - \mu) .\ p_i$

$\sigma = \Sigma\, p_i\, x_i - \mu$ (Note that Standard Deviation (SD) = $+\sqrt{\sigma^2}$)

(d) The probability distribution for a binomial variate 'X' is given by ; $P(X = r) = {}^nC_r\, p^r q^{n-r}$ where : p = probability of success in a single trial, q = probability of failure in a single trial and p + q = 1. The recurrence formula $\frac{P(r+1)}{P(r)} = \frac{n-r}{r+1}\cdot\frac{p}{q}$, is very helpful for quickly computing P(1), P(2). P(3) etc. if P(0) is known.

(e) Mean of Binomial Probability Distribution (BPD) = np ; variance of BPD = npq.

(f) If p represents a person chance of success in any venture and 'M' the sum of money which he will receive in case of success, then his expectations or probable value = pM

Expectations = pM

15. GEOMETRICAL PROBABILITY :

The following statements are axiomatic :

(a) If a point is taken at random on a given straight line AB, the chance that it falls on a particular segment PQ of the line is PQ/AB.

(b) If a point is taken at random on the area S which includes an area σ, the chance that the point falls on σ is σ/S.

16. IMPORTAINT POINTS :

(a) If $A_1 \subseteq A_2$ then $P(A_1) \le P(A_2)$ and $P(A_2 - A_1) = P(A_2) - P(A_1)$

(b) If $A = A_1 \cup A_2 \cup .... \cup A_n$ where $A_1, A_2....A_n$ are mutually exclusive events then

$P(A) = P(A_1) + P(A_2) + ...P(A_n)$

(c) Let A & B are two events corresponding to sample space S then P(S/A) = P(A/A) =1

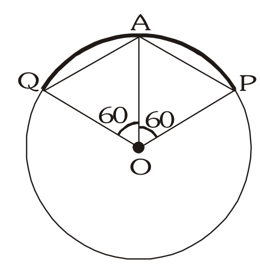
(d) Let A and B are two events corresponding to sample space S and F is any other event s.t. $P(F) \ne 0$ then $P((A \cup B)/F) = P(A/F) + P(B/F) - P((A \cap B)/F)$

(e) P(A'/B) = 1 – P(A/B)

(f) $P(A \cap B) \le P(A), P(B) \le P(A \cup B) \le P(A) + P(B)$

Illustration 36 : If two points are selected at random on the circumference of a circle, find the probability that their distance apart is less than the radius of the circle.

Solution : Let one of the selected points be A, which has been selected as shown in the figure. Now, for the selection of second point, the point must lie on the thick arc QAP, since ΔOPA and ΔOQA are equilateral triangle with side r.



$\Rightarrow$ Probability of selecting the second point $= \dfrac{r \times \frac{2\pi}{3}}{2\pi r} = \dfrac{1}{3}$

Illustration 37 : A wire of length ℓ is cut into three pieces. Find the probability that the three pieces form a triangle.

Solution : Let the lengths of three parts of the wire be x,y and $\ell - (x + y)$, then $x > 0$, $y > 0$, $\ell - (x + y) > 0$ i.e. $x + y < \ell$

Since in a triangle, the sum of any two sides is greater than third side, so

$$x + y > \ell - (x + y) \Rightarrow x + y > \frac{\ell}{2}$$

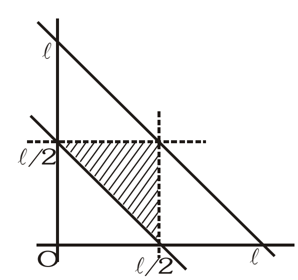
$$x + \ell - (x + y) > y \Rightarrow y < \frac{\ell}{2}$$

$$y + \ell - (x + y) > x \Rightarrow x < \frac{\ell}{2}$$

$\Rightarrow$ Favourable cases : $x + y > \dfrac{\ell}{2}$; $y < \dfrac{\ell}{2}$; $x < \dfrac{\ell}{2}$

& Total cases : $x + y < \ell$; $x > 0$; $y > 0$

$$P(E) = \frac{\text{Favourable Area}}{\text{Total Area}} = \frac{\ell^2/8}{\ell^2/2} = \frac{1}{4}$$



Do yourself - 11 :

(i) A point is selected at random inside a circle. The probability that the point is closer to the center of the circle than to its circumference is -

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{\sqrt{2}}$

Miscellaneous Illustrations :

Illustration 38 : Three persons A, B, C in order cut a pack of cards, replacing them after each cut, on the condition that the first who cuts a spade shall win a prize; find their respective chances. [REE 1992]

Solution : Let p be the chance of cutting a spade and q the chance of not cutting a spade from a pack of 52 cards.

Then $p = \frac{^{13}C_1}{^{52}C_1} = \frac{1}{4}$ and $q = 1 - \frac{1}{4} = \frac{3}{4}$

Now A will win a prize if he cuts spade at 1st, 4th, 7th, 10th turns, etc. Note that A will get a second chance if A, B, C all fail to cut a spade once and then A cuts a spade at the 4th turn. Similarly he will cut a spade at the 7th turn when A, B, C fail to cut spade twice, etc.

Hence A's chance of winning the prize $= p + q^3p + q^6p + q^9p + ... = \frac{p}{1-q^3} = \frac{\frac{1}{4}}{1-\left(\frac{3}{4}\right)^3} = \frac{16}{37}$

Similarly B's chance $= (qp + q^4p + q^7p + ...) = q(p + q^3p + q^6p + ...) = \frac{3}{4}.\frac{16}{37} = \frac{12}{37}$

and C's chance $= \frac{3}{4}$ of B's chance $= \frac{3}{4}.\frac{12}{37} = \frac{9}{37}$

Illustration 39 : (a) If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with replacement, determine the probability that the roots of the equation $x^2 + px + q = 0$ are real.

[IIT 1997]

(b) Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the equation will have equal roots. [REE 1998]

Solution : (a) If roots of $x^2 + px + q = 0$ are real, then $p^2 - 4q \geq 0$ .... (i)

Both p, q belongs to set S = {1, 2, 3, ... 10} when p = 1, no value of q from S will satisfy (i)

p = 2	q = 1 will satisfy	1 value
p = 3	q = 1, 2	2 value
p = 4	q = 1, 2, 3, 4	4 value
p = 5	q = 1, 2, 3, 4, 5, 6	6 value
p = 6	q = 1, 2, 3, 4, 5, 6, 7, 8, 9	9 value

For p = 7, 8, 9, 10 all the ten values of q will satisfy.

Sum of these selections is 1 + 2 + 4 + 6 + 9 + 10 + 10 + 10 + 10 = 62

But the total number of selections of p and q without any order is $10 \quad 10 = 100$

Hence the required probability is $= \frac{62}{100} = 0.62$

(b) Roots equal $\Rightarrow$ $b^2 - 4ac = 0$

$$\therefore \quad \left(\frac{b}{2}\right)^2 = ac \qquad \text{...... (i)}$$

Each coefficient is an integer, so we consider the following cases :

$b = 1 \qquad \therefore \quad \frac{1}{4} = ac$

No integral values of a and c

$b = 2 \qquad 1 = ac \qquad \therefore \quad (1, 1)$

$b = 3 \qquad 9/2 = ac$

No integral values of a and c

$b = 4 \qquad 4 = ac \qquad \therefore \quad (1, 4), (2, 2), (4, 1)$

$b = 5 \qquad 25/2 = ac$

No integral values of a and c

$b = 6 \qquad 9 = ac \qquad \therefore \quad (3, 3)$

Thus we have 5 favourable way for b = 2, 4, 6

Total number of equations is 6.6.6 = 216

$\therefore$ Required probability is $\frac{5}{216}$

Illustration 40 : A set A has n elements. A subset P of A is selected at random. Returning the element of P, the set Q is formed again and then a subset Q is selected from it. Find the probability that P and Q have no common elements. **[IIT 1990]**

Solution : The set P be the empty set, or one element set or two elements set .... or n elements set. Then the set Q will be chosen from amongst the remaining n elements or n – 1 elements or n – 2 elements .... or no elements. The probability of P being an empty set is ${}^nC_0/2^n$, the probability of P being one element set is ${}^nC_1/2^n$ and in general, the probability of P being an r element set is ${}^nC_r/2^n$.

When the set P consisting of r elements is chosen from A, then the probability of choosing the set Q from amongst the remaining n – r elements is $2^{n-r}/2^n$. Hence the probability that P and Q have no common elements is given by

$$\sum_{r=0}^{n} \frac{{}^nC_r}{2^n} \cdot \frac{2^{n-r}}{2^n} = \frac{1}{4^n} \sum_{r=0}^{n} {}^nC_r . 2^{n-r} = \left(\frac{1}{4}\right)^n (1+2)^n = \left(\frac{3}{4}\right)^n \qquad \text{[By binomial theorem]}$$

Illustration 41 : The probabilities of three events A, B and C are P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$, find $P(B \cap C)$. [REE 1996]

Solution : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$\therefore \quad P(A \cap B) = 0.2$

Now $P(A \cup B \cup C) = S_1 - S_2 + S_3 = (0.6 + 0.4 + 0.5) - (0.2 + P(B \cap C) + 0.3) + 0.2$
$= 1.5 - 0.3 - P(B \cap C)$

We know $0.85 \leq P(A \cup B \cup C) \leq 1$

or $\quad 0.85 \leq 1.2 - P(B \cap C) \leq 1$

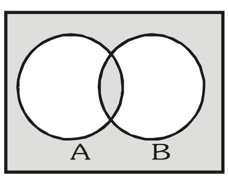
$\therefore \quad 0.2 \leq P(B \cap C) \leq 0.35$

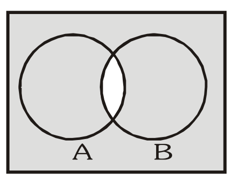
ANSWERS FOR DO YOURSELF

1 : (i) $\{B_1R_1, B_2R_1, B_3R_1, B_1R_2, B_2R_2, B_3R_2\}$ (ii) $\{M_1W_1, M_2W_1, M_1W_2, M_2W_2, M_1W_3, M_2W_3\}$

(iii) {H1, H2, H3, H4, H5, H6, TH,TT} (iv) A

2 : (i) 5/12 (ii) 3:10, 3 : 1 (iii) 1/10 (iv) $\dfrac{(13)^5 - (12)^5}{(13)^5}$

3 : (i) (a) 

(b) 

(ii) A,B,D (iii) 11/15

(iv) (a) $\dfrac{19}{25}$ (b) $\dfrac{13}{25}$

4 : (i) A (ii) 1/5525

5 : (i) A, B (ii) 1/36

6 : (i) 2/21 (ii) $\dfrac{1}{n+2}$

7 : (i) (a) 0.72 (b) 0.23

8 : (i) $\dfrac{496}{729}$ (ii) ${}^7C_3\left(\dfrac{1}{6}\right)^3\left(\dfrac{5}{6}\right)^4$

9 : (i) 2/3 (ii) 9/25

10 : (i) $\dfrac{7}{16}$ (ii) (a) $\dfrac{2}{5}$ (b) $\dfrac{13}{25}$

11 : (i) A

EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. A quadratic equation is chosen from the set of all quadratic equations which are unchanged by squaring their roots. The chance that the chosen equation has equal roots is -

(A) 1/2 (B) 1/3 (C) 1/4 (D) 2/3

2. 5 persons entered the lift cabin on the ground floor of an 8 floor building. Suppose that each of them independently and with equal probability, can leave the cabin at any other floor, starting from the first. The probability that all 5 persons leave at different floors is -

(A) $\left(\frac{5}{8}\right)^5$ (B) $\frac{{}^8C_5}{8^5}$ (C) $\frac{5!}{8^5}$ (D) $\frac{{}^8C_5 5!}{8^5}$

3 If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals - [JEE 99]

(A) $\frac{1}{4}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{1}{49}$

4. There are ten prizes, five A's, three B's and two C's, placed in identical sealed envelopes for the top ten contestants in a mathematics contest. The prizes are awarded by allowing winners to select an envelope at random from those remaining. When the 8^{th} contestant goes to select the prize, the probability that the remaining three prizes are one A, one B and one C, is -

(A) 1/4 (B) 1/3 (C) 1/12 (D) 1/10

5. A & B are two independent events such that $P(\bar{A}) = 0.7$, $P(\bar{B}) = a$ & $P(A \cup B) = 0.8$, then a =

(A) 5/7 (B) 2/7 (C) 1 (D) none

6. A determinant is chosen at random from the set of all determinant of order 2 with elements 0 or 1 only. The probability that the determinant chosen has the value non negative is -

(A) 3/16 (B) 6/16 (C) 10/16 (D) 13/16

7. A license plate is 3 letters (of English alphabets) followed by 3 digits. If all possible license plates are equally likely, the probability that a plate has either a letter palindrome or a digit palindrome (or both), is -

(A) $\frac{7}{52}$ (B) $\frac{9}{65}$ (C) $\frac{8}{65}$ (D) none

8. Two cubes have their faces painted either red or blue. The first cube has five red faces and one blue face. When the two cubes are rolled simultaneously, the probability that the two top faces show the same colour is 1/2. Number of red faces on the second cube, is -

(A) 1 (B) 2 (C) 3 (D) 4

9. A is one of the 6 horses entered for a race and is to be ridden by one of two jockeys B or C. It is 2 to 1 that B rides A, in which case all the horses are equally likely to win, if C rides A, his chance is trebled, Then the odds against his winning are -

(A) $\frac{5}{13}$ (B) $\frac{18}{5}$ (C) $\frac{13}{18}$ (D) $\frac{13}{5}$

10. Lot A consists of 3G and 2D articles. Lot B consists of 4G and 1D article. A new lot C is formed by taking 3 articles from A and 2 from B. The probability that an article chosen at random from C is defective, is -

(A) 1/3 (B) 2/5 (C) 8/25 (D) none

11. 'A' and 'B' each have a bag that contains one ball of each of the colours blue, green, orange, red and violet. 'A' randomly selects one ball from his bag and puts it into B's bag. 'B' then randomly selects one ball from his bag and puts it into A's bag. The probability that after this process the contents of the two bags are the same, is -

(A) 1/2 (B) 1/3 (C) 1/5 (D) 1/6

12. A bowl has 6 red marbles and 3 green marbles. The probability that a blind folded person will draw a red marble on the second draw from the bowl without replacing the marble from the first draw, is -

(A) 2/3 (B) 1/4 (C) 5/12 (D) 5/8

13. Two cards are drawn from a well shuffled pack of 52 playing cards one by one. If

A : the event that the second card drawn is an ace and

B : the event that the first card drawn is an ace card.

then which of the following is true ?

(A) $P(A) = \frac{4}{17}$; $P(B) = \frac{1}{13}$ (B) $P(A) = \frac{1}{13}$; $P(B) = \frac{1}{13}$

(C) $P(A) = \frac{1}{13}$; $P(B) = \frac{1}{17}$ (D) $P(A) = \frac{16}{221}$; $P(B) = \frac{4}{51}$

14. An Urn contains 'm' white and 'n' black balls. All the balls except for one ball, are drawn from it. The probability that the last ball remaining in the Urn is white, is -

(A) $\frac{m}{m+n}$ (B) $\frac{n}{m+n}$ (C) $\frac{1}{(m+n)!}$ (D) $\frac{mn}{(m+n)!}$

15. Three numbers are chosen at random without replacement from {1, 2, 3,....., 10}. The probability that the minimum of the chosen numbers is 3 or their maximum is 7 is -

(A) 1/2 (B) 1/3 (C) 1/4 (D) 11/40

16. If atleast one child in a family with 3 children is a boy then the probability that 2 of the children are boys, is -

(A) $\frac{3}{7}$ (B) $\frac{4}{7}$ (C) $\frac{1}{3}$ (D) $\frac{3}{8}$

17. 7 persons are stopped on the road at random and asked about their birthdays. If the probability that 3 of them are born on Wednesday, 2 on Thursday and the remaining 2 on Sunday is $\frac{K}{7^6}$, then K is equal to -

(A) 15 (B) 30 (C) 105 (D) 210

18. A box has four dice in it. Three of them are fair dice but the fourth one has the number five on all of its faces. A die is chosen at random from the box and is rolled three times and shows up the face five on all the three occasions. The chance that the die chosen was a rigged die, is -

(A) $\frac{216}{217}$ (B) $\frac{215}{219}$ (C) $\frac{216}{219}$ (D) none

19. Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian and will mistake it for a French wine with probability 0.2. Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine and solemnly says : "French". The probability that the wine he tasted was Californian, is nearly equal to -

(A) 0.14 (B) 0.24 (C) 0.34 (D) 0.44

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

20. For two events A & B, which of the following is/are correct -

(A) $(A \cup B) \cap B = B$ (B) $(A \cup B)^C \cap B^C = (A \cup B)^C$

(C) $A^C \cap (A \cap B) \cap B^C = \phi$ (D) $(A^C \cap B) \cap (A \cup B) = B$

21. From a pack of 52 playing cards, face cards and tens are removed and kept aside then a card is drawn at random from the remaining cards. If

A : The event that the card drawn is an ace

H : The event that the card drawn is a heart

S : The event that the card drawn is a spade

then which of the following holds ?

(A) $9P(A) = 4P(H)$ (B) $P(S) = 4P(A \cap H)$ (C) $4P(H) = 3P(A \cup S)$ (D) $P(H) = 9P(A \cap S)$

22. Before a race the chance of three runners A, B & C were estimated to be proportional to 5, 3 & 2 respectively but during the race A meets with an accident which reduces his chance to 1/3. If the respective chances of B and C are P(B) and P(C) then -

(A) $P(B) = \frac{2}{5}$ (B) $P(C) = \frac{4}{15}$ (C) $P(C) = \frac{2}{5}$ (D) $P(B) = \frac{4}{15}$

23. If E & F are events with $P(E) \le P(F)$ & $P(E \cap F) > 0$, then - [JEE 98]

(A) occurrence of E $\Rightarrow$ occurrence of F . (B) occurrence of F $\Rightarrow$ occurrence of E

(C) non – occurrence of E $\Rightarrow$ non – occurrence of F (D) none of the above implications holds.

24. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{5}$, then -

(A) $P(A \cup B) = \frac{3}{5}$ (B) $P(A / B) = \frac{1}{2}$

(C) $P(A / A \cup B) = \frac{5}{6}$ (D) $P(A \cap B / A' \cup B') = 0$

CHECK YOUR GRASP — ANSWER KEY — EXERCISE-01

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	D	A	A	B	D	A	C	D	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	A	B	A	D	A	B	C	C	A,B,C
Que.	21	22	23	24						
Ans.	A,C,D	A,B	D	A,B,C,D						

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. If two of the 64 squares are chosen at random on a chess board, the probability that they have a side in common is -

(A) 1/9 (B) 1/18 (C) 2/7 (D) none of these

2. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\ P(B)$. Then -

(A) $P\left(\frac{B}{A}\right) = P(B) - P(A)$ (B) $P(A^C \cup B^C) = P(A^C) + P(B^C)$

(C) $P((A \cup B)^C) = P(A^C)\ P(B^C)$ (D) $P\left(\frac{A}{B}\right) = P(A)$

3. 15 coupons are numbered 1,2,3,.........,15 respectively. 7 coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9 is -

(A) $\left(\frac{9}{16}\right)^6$ (B) $\left(\frac{8}{15}\right)^7$ (C) $\left(\frac{3}{5}\right)^7$ (D) $\frac{9^7 - 8^7}{15^7}$

4. A child throws 2 fair dice. If the numbers showing are unequal, he adds them together to get his final score. On the other hand, if the numbers showing are equal, he throws 2 more dice & adds all 4 numbers showing to get his final score. The probability that his final score is 6 is -

(A) $\frac{145}{1296}$ (B) $\frac{146}{1296}$ (C) $\frac{147}{1296}$ (D) $\frac{148}{1296}$

5. If E_1 and E_2 are two events such that $P(E_1)=1/4$, $P(E_2/E_1)=1/2$ and $P(E_1/E_2) = 1/4$ then -

(A) E_1 and E_2 are independent

(B) E_1 and E_2 are exhaustive

(C) E_2 is twice as likely to occur as E_1

(D) probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in G.P.

6. Two numbers a and b are selected from the set of natural number then the probability that $a^2 + b^2$ is divisible by 5 is -

(A) $\frac{9}{25}$ (B) $\frac{7}{18}$ (C) $\frac{11}{36}$ (D) $\frac{17}{81}$

7. If a, b and c are three numbers (not necessarily different) chosen randomly and with replacement from the set {1, 2, 3, 4, 5}, the probability that (ab + c) is even, is -

(A) $\frac{50}{125}$ (B) $\frac{59}{125}$ (C) $\frac{64}{125}$ (D) $\frac{75}{125}$

8. For any two events A & B in a sample space :

(A) $P\left(\frac{A}{B}\right) \geq \frac{P(A) + P(B) - 1}{P(B)}$, $P(B) \neq 0$ is always true

(B) $P\left(A \cap \overline{B}\right) = P(A) - P(A \cap B)$

(C) $P(A \cup B) = 1 - P\left(A^C\right) P\left(B^C\right)$, if A & B are independent

(D) $P(A \cup B) = 1 - P\left(A^C\right) P\left(B^C\right)$, if A & B are disjoint

9. In a horse race there are 18 horses numbered from 1 to 18. The probability that horse 1 would win is $\frac{1}{6}$, horse 2 is $\frac{1}{10}$ and 3 is $\frac{1}{8}$. Assuming a tie is impossible, the chance that one of the three horses wins the race, is -

(A) $\frac{143}{420}$ (B) $\frac{119}{120}$ (C) $\frac{47}{120}$ (D) $\frac{1}{5}$

10. The probability that a radar will detect an object in one cycle is p. The probability that the object will be detected in n cycles is -

(A) $1-p^n$ (B) $1-(1-p)^n$ (C) p^n (D) $p(1-p)^{n-1}$

11. Two real numbers, x & y are selected at random. Given that $0 \le x \le 1;\ 0 \le y \le 1$. Let A be the event that $y^2 \le x$; B be the event that $x^2 \le y$, then -

(A) $P(A \cap B) = \frac{1}{3}$ (B) A & B are exhaustive events

(C) A & B are mutually exclusive (D) A & B are independent events.

12. A Urn contains 'm' white and 'n' black balls. Balls are drawn one by one till all the balls are drawn. Probability that the second drawn ball is white, is -

(A) $\frac{m}{m+n}$ (B) $\frac{m(n-1)}{(m+n)(m+n-1)}$

(C) $\frac{m(m-1)}{(m+n)(m+n-1)}$ (D) $\frac{mn}{(m+n)(m+n-1)}$

13. Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is 1/5. The probability that bus B will be late is 7/25. The probability that the bus B is late given that bus A is late is 9/10. Then the probabilities

(i) neither bus will be late on a particular day and

(ii) bus A is late given that bus B is late, are respectively

(A) 2/25 and 12/28 (B) 18/25 and 22/28 (C) 7/10 and 18/28 (D) 12/25 and 2/28

14. If A & B are two events such that $P(B) \neq 1$, B^C denotes the event complementary to B, then -

(A) $P(A/B^C) = \frac{P(A)-P(A \cap B)}{1-P(B)}$ (B) $P(A \cap B) \ge P(A) + P(B) - 1$

(C) $P(A) \gtrless P(A/B)$ according as $P(A/B^C) \gtrless P(A)$ (D) $P(A/B^C) + P(A^C/B^C) = 1$

15. The probabilities of events, $A \cap B$, A, B & $A \cup B$ are respectively in A.P. with probability of second term equal to the common difference. Therefore the events A and B are -

(A) compatible (B) independent

(C) such that one of them must occur (D) such that one is twice as likely as the other

16. From an urn containing six balls, 3 white and 3 black ones, a person selects at random an even number of balls (all the different ways of drawing an even number of balls are considered equally probable, irrespective of their number). Then the probability that there will be the same number of black and white balls among them -

(A) $\frac{4}{5}$ (B) $\frac{11}{15}$ (C) $\frac{11}{30}$ (D) $\frac{2}{5}$

17. A pair of fair dice having six faces numbered from 1 to 6 are thrown once, suppose two events E and F are defined as -

E : Product of the two numbers appearing is divisible by 5.

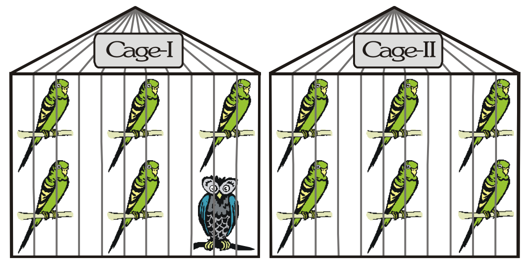
F : At least one of the dice shows up the face one.

Then the events E and F are

(A) mutually exclusive (B) independent

(C) neither independent nor mutually exclusive (D) are equiprobable

18. Shalu brought two cages of birds : Cage-I contains 5 parrots and 1 owl and Cage-II contains 6 parrots, as shown. One day Shalu forgot to lock both cages and two birds flew from Cage-I to Cage-II. Then two birds flew back from Cage-II to Cage-I. Assume that all birds have equal chance of flying, the probability that the Owl is still in Cage-I, is -



Birds like to fly

(A) 1/6 (B) 1/3

(C) 2/3 (D) 3/4

19. In a maths paper there are 3 sections A, B & C. Section A is compulsory. Out of sections B & C a student has to attempt any one. Passing in the paper means passing in A & passing in B or C. The probability of the student passing in A, B & C are p, q & 1/2 respectively. If the probability that the student is successful is 1/2 then, which of the following is false -

(A) $p = q = 1$ (B) $p = q = 1/2$ (C) $p = 1, q = 0$ (D) $p = 1, q = 1/2$

20. Sixteen players $s_1, s_2, \ldots\ldots, s_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. The probability that "exactly one of the two players s_1 & s_2 is among the eight winners" is -

(A) $\frac{4}{15}$ (B) $\frac{7}{15}$ (C) $\frac{8}{15}$ (D) $\frac{9}{15}$

21. The number 'a' is randomly selected from the set {0, 1, 2, 3,.........98, 99}. The number 'b' is selected from the same set. Probability that the number $3^a + 7^b$ has a digit equal to 8 at the units place, is -

(A) $\frac{1}{16}$ (B) $\frac{2}{16}$ (C) $\frac{4}{16}$ (D) $\frac{3}{16}$

22 If $\bar{E}$ & $\bar{F}$ are the complementary events of events E & F respectively & if $0 < P(F) < 1$, then - **[JEE 98]**

(A) $P(E|F) + P(\bar{E}|F) = 1$ (B) $P(E|F) + P(E|\bar{F}) = 1$ (C) $P(\bar{E}|F) + P(E|\bar{F}) = 1$ (D) $P(E|\bar{F}) + P(\bar{E}|\bar{F}) = 1$

BRAIN TEASERS **ANSWER KEY** EXERCISE-02

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C,D	D	D	A,C,D	A	B	A,B,C	C	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A,B	A	C	A,B,C,D	D	B	C,D	D	A,B,C	C
Que.	21	22								
Ans.	D	A,D								

EXERCISE - 03

MISCELLANEOUS TYPE QUESTIONS

FILL IN THE BLANKS :

1. If $P(A \cup B) = P(A \cap B)$ then the relation between P(A) and P(B) is ....
2. Let A and B be two events such that P(A) = 0.3 and $P(A \cup B) = 0.8$. If A and B are independent events then P(B) = ...
3. Three faces of a fair die are yellow, two faces red and one blue. The die is tossed three times. The probability that the colours yellow, red and blue, appear in the first, second and the third tosses respectively is ....
4. If $\frac{(1+3p)}{3}$, $\frac{(1-p)}{4}$ and $\frac{(1-2p)}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is ....
5. The probability that a randomly selected three-digit number has exactly three factors will be ....

MATCH THE COLUMN :

Following questions contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1.

	Column-I		Column-II
(A)	A natural number x is randomly selected from the set of first 100 natural numbers. The probability that it satisfies the inequality $x+\frac{100}{x}>50$ is	(p)	$\frac{12}{17}$
(B)	5 different marbles are placed in 5 different boxes randomly. If each box can hold any number of marbles then the probability that exactly two boxes remain empty is	(q)	$\frac{11}{20}$
(C)	A letter is known to have come either from London or Clifton. On the postmark only the two consecutive letters ON are legible. The chance that it came from London is	(r)	$\frac{12}{25}$
(D)	There are three works, one consisting of 3 volumes, one of 4 and the other of one volume. They are placed on a shelf at random. If the chance that volumes of same works are all together is P_1 then $7P_1$ =	(s)	$\frac{3}{20}$

2. An urn contain six red balls and four black balls. All ten balls are drawn from the urn, one by one and their colour is noted. Balls are not replaced once they have been drawn.

	Column-I		Column-II
(A)	Probability that first three balls are of same colour	(p)	$\frac{1}{5}$
(B)	Probability that last three balls are of same colour	(q)	$\frac{3}{5}$
(C)	If it is known that first three are of the same colour, then the probability that colour is red is	(r)	$\frac{4}{15}$
(D)	Probability that no two consecutive balls in first three draw are same is	(s)	$\frac{5}{6}$

ASSERTION & REASON

These questions contain, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I

(C) Statement-I is true, Statement-II is false

(D) Statement-I is false, Statement-II is true

1. Statement-I : $P\left(\frac{(A \cap \overline{B})}{C}\right) = P\left(\frac{A}{C}\right) + P\left(\frac{A \cap B}{C}\right)$

Because

Statement-II : $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

(A) A (B) B (C) C (D) D

2. From a well shuffled pack of 52 playing cards a card is drawn at random. Two events A and B are defined as :

A : Red card is drawn ; B : Card drawn is either a Diamond or Heart

Statement-I : P(A+B) = P(AB)

Because

Statement-II : $A \subseteq B$ and $B \subseteq A$

(A) A (B) B (C) C (D) D

3. A fair coin is tossed 3 times. Consider the events

A : first toss is head ; B : second toss is head ;

C : exactly two consecutive heads or exactly two consecutive tails

Statement-I : A,B,C are independent events.

Because

Statement-II : A,B,C are pairwise independent.

(A) A (B) B (C) C (D) D

4. Consider (a, b), where a and b are respective outcomes in throwing an unbiased die twice.

Statement-I : If $\lim_{x\to 0}\left(\frac{a^x + b^x + 1}{3}\right)^{\frac{3}{x}} = 6$, then the probability that 'a' is a prime number is $\frac{1}{2}$.

Because

Statement-II : If A & B are two events then probability of event B when event A has already happened is $P(B/A) = \frac{P(A \cap B)}{P(A)}$.

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

Let S and T are two events defined on a sample space with probabilities

P(S) = 0.5, P(T) = 0.69, P(S/T) = 0.5

On the basis of above information, answer the following questions :

1. Events S and T are -

(A) mutually exclusive (B) independent

(C) mutually exclusive and independent (D) neither mutually exclusive nor independent

2. The value of P(S and T) -

(A) 0.3450 (B) 0.2500 (C) 0.6900 (D) 0.350

3. The value of P(S or T) -

(A) 0.6900 (B) 1.19 (C) 0.8450 (D) 0

Comprehension # 2

A JEE aspirant estimates that she will be successful with an 80 percent chance if she studies 10 hours per day, with a 60 percent chance if she studies 7 hours per day and with a 40 percent chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively.

On the basis of above information, answer the following questions :

1. The chance she will be successful, is -

(A) 0.28 (B) 0.38 (C) 0.48 (D) 0.58

2. Given that she is successful, the chance she studied for 4 hours, is -

(A) $\frac{6}{12}$ (B) $\frac{7}{12}$ (C) $\frac{8}{12}$ (D) $\frac{9}{12}$

3. Given that she does not achieve success, the chance she studied for 4 hour, is -

(A) $\frac{18}{26}$ (B) $\frac{19}{26}$ (C) $\frac{20}{26}$ (D) $\frac{21}{26}$

MISCELLANEOUS TYPE QUESTION — **ANSWER KEY** — EXERCISE-03

- ***Fill in the Blanks***

1. P(A) = P(B) **2.** 5/7 **3.** 1/36 **4.** $\frac{1}{3} \le p \le \frac{1}{2}$ **5.** $\frac{7}{900}$

- ***Match the Column***

1. (A) → (q); (B) → (r); (C) → (p); (D) → (s) **2.** (A) → (p); (B) → (p); (C) → (s); (D) → (r)

- ***Assertion & Reason***

1. D **2.** A **3.** B **4.** A

- ***Comprehension Based Questions***

Comprehension # 1 : **1.** B **2.** A **3.** C

Comprehension # 2 : **1.** C **2.** B **3.** D

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

1. Numbers are selected at random, one at a time, from the two digit numbers 00, 01, 02,....., 99 with replacement. An event E occurs if & only if the product of the two digits of a selected number is 18. If four numbers are selected, find the probability that the event E occurs at least 3 times.
2. A pair of fair dice is tossed. Find the probability that the maximum of the two numbers is greater than 4.
3. In a given race, the odds in favour of four horses A, B, C & D are 1 : 3, 1 : 4, 1 : 5 and 1 : 6 respectively. Assuming that a dead heat is impossible, find the chance that one of them wins the race.
4. What is the probability that in a group of
   (a) 2 people, both will have the same date of birth.
   (b) 3 people, atleast 2 will have the same date of birth.
   Assume the year to be ordinary consisting of 365 days.
5. The odds that a book will be favourably reviewed by three independent critics are 5 to 2, 4 to 3 and 3 to 4 respectively. What is the probability that of the three reviews a majority will be favourable?
6. In a box, there are 8 alphabets cards with the letters : S, S, A, A, A, H, H, H. Find the probability that the word 'ASH' will form if :
   (a) the three cards are drawn one by one & placed on the table in the same order that they are drawn.
   (b) the three cards are drawn simultaneously.
7. There are 2 groups of subjects one of which consists of 5 science subjects & 3 engg. subjects & other consists of 3 science & 5 engg. subjects. An unbiased die is cast. If the number 3 or 5 turns up a subject is selected at random from first group, otherwise the subject is selected from 2^{nd} group. Find the probability that an engg. subject is selected.
8. In a building programme the event that all the materials will be delivered at the correct time is M and the event that the building programme will be completed on time is F. Given that P(M) = 0.8 and P(M $\cap$ F) = 0.65, find P(F/M). If P(F) = 0.7, find the probability that the building programme will be completed on time if all the materials are not delivered at the correct time.
9. A covered basket of flowers has some lilies and roses. In search of rose, Sweety and Shweta alternately pick up a flower from the basket but put it back if it is not a rose. Sweety is 3 times more likely to be the first one to pick a rose. If sweety begin this 'rose hunt' and if there are 60 lilies in the basket, find the number of roses in the basket.
10. The probability that an archer hits the target when it is windy is 0.4; when it is not windy, her probability of hitting the target is 0.7. On any shot, the probability of a gust of wind is 0.3. Find the probability that :
    (a) She hits the target on first shot
    (b) She hits the target exactly once in two shots.
11. A room has three electric lamps. From a collection of 10 electric bulbs of which 6 are good 3 are selected at random & put in the lamps. Find the probability that the room is lighted.
12. A bomber wants to destroy a bridge. Two bombs are sufficient to destroy it. If four bombs are dropped, what is the probability that it is destroyed, if the chance of a bomb hitting the target is 0.4.
13. The chance of one event happening is the square of the chance of a 2^{nd} event, but odds against the first are the cubes of the odds against the 2^{nd}. Find the chances of each. (Assume that both events are neither sure nor impossible).
14. A box contains 5 radio tubes of which 2 are defective. The tubes are tested one after the other until the 2 defective tubes are discovered. Find the probability that the process stopped on the (a) Second test ; (b) Third test. If the process stopped on the third test, find the probability that the first tube is non defective.
15. Anand plays with Karpov 3 games of chess. The probability that he wins a game is 0.5, looses with probability 0.3 and ties with probability 0.2. If he plays 3 games then find the probability that he wins atleast two games.
16. An aircraft gun can take a maximum of four shots at an enemy's plane moving away from it. The probability of hitting the plane at first, second, third & fourth shots are 0.4, 0.3, 0.2 & 0.1 respectively. What is the probability that the gun hits the plane.

17. One hundred management students who read at least one of the three business magazines are surveyed to study the readership pattern. It is found that 80 read Business India, 50 read Business World and 30 read Business Today. Five students read all the three magazines. A student was selected randomly. Find the probability that he reads exactly two magazines.

18. An author writes a good book with a probability of 1/2. If it is good it is published with a probability of 2/3. If it is not, it is published with a probability of 1/4. Find the probability that he will get atleast one book published if he writes two.

19. 3 students {A, B, C} tackle a puzzle together and offers a solution upon which majority of the 3 agrees. Probability of A solving the puzzle correctly is p. Probability of B solving the puzzle correctly is also p. C is a dumb student who randomly supports the solution of either A or B. There is one more student D, whose probability of solving the puzzle correctly is once again, p. Out of the 3 member team {A, B, C} and one member team {D}, which one is more likely to solve the puzzle correctly.

20. Consider the following events for a family with children A = {of both sexes}; B = {at most one boy}
In which of the following (are/is) the events A and B are independent.
(a) if a family has 3 children (b) if a family has 2 children
Assume that the birth of boy or a girl is equally likely mutually exclusive and exhaustive.

21. Each of the 'n' passengers sitting in a bus may get down from it at the next stop with probability p. Moreover, at the next stop either no passenger or exactly one passenger boards the bus. The probability of no passenger boarding the bus at the next stop being p_o. Find the probability that when the bus continues on its way after the stop, there will again be 'n' passengers in the bus.

22. An examination consists of 8 questions in each of which the candidate must say which one of the 5 alternatives is correct one. Assuming that the student has not prepared earlier chooses for each of the question any one of 5 answers with equal probability.
(a) prove that the probability that he gets more than one correct answer is $(5^8 - 3 \times 4^8) / 5^8$.
(b) find the probability that he gets correct answers to six or more questions.

23. A purse contains n coins of unknown value, a coin drawn at random is found to be a rupee, what is the chance that it is the only rupee in the purse? Assume all numbers of rupee coins in the purse to be equally likely.

24. A biased coin which comes up heads three time as often as tails is tossed. If it shows head, a chip is drawn from urn-I which contains 2 white chips and 5 red chips. If the coin comes up tail, a chip is drawn from urn-II which contains 7 white and 4 red chips. Given that a red chip was drawn, what is the probability that the coin came up head ?

25. 3 players A, B & C toss a coin cyclically in that order (that is A, B, C, A, B, C, A, B, ...) till a head shows. Let p be the probability that the coin shows a head. Let α, β & γ be respectively the probability that A, B and C gets the first head. Prove that $\beta = (1-p)\alpha$. Determine α, β & γ (in terms of p). **[JEE 98]**

CONCEPTUAL SUBJECTIVE EXERCISE — ANSWER KEY — EXERCISE-04(A)

1. $97/(25)^4$ **2.** 5/9 **3.** 319/420 **4.** (a) $\frac{1}{365}$; (b) $1-\frac{364 \times 363}{(365)^2}$ **5.** $\frac{209}{343}$

6. (a) 3/56 (b) 9/28 **7.** 13/24 **8.** $P(F/M)=\frac{13}{16}$; $P(F/\bar{M})=\frac{1}{4}$ **9.** 120

10. (a) 0.61; (b) 0.4758 **11.** $\frac{29}{30}$ **12.** $\frac{328}{625}$ **13.** $\frac{1}{9}, \frac{1}{3}$ **14.** (a) 1/10 (b) 3/10 (c) 2/3

15. 1/2 **16.** 0.6976 **17.** 1/2 **18.** 407/576 **19.** Both are equally likely

20. Independent in (a) and not independent in (b) **21.** $(1-p)^{n-1} [p_o(1-p) + np(1-p_0)]$

22. $\frac{481}{5^8}$ **23.** $\frac{2}{n(n+1)}$ **24.** $\frac{165}{193}$ **25** $\alpha = \frac{p}{1-(1-p)^3}$, $\beta = \frac{(1-p)\ p}{1-(1-p)^3}$, $\gamma = \frac{(1-p)^2 p}{1-(1-p)^3}$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. Two distinct numbers a and b are chosen randomly from the set $\{2, 2^2, 2^3, 2^4, \ldots, 2^{25}\}$. Find the probability that $\log_a b$ is an integer.

2. A uniform unbiased die is constructed in the shape of a regular tetrahedron with faces numbered 2, 2, 3 and 4 and the score is taken from the face on which the die lands. If two such dice are thrown together, find the probability of scoring.

(a) exactly 6 on each of 3 successive throws.

(b) more than 4 on atleast one of the three successive throws.

3. A cube with all six faces coloured is cut into 64 cubical blocks of the same size which are thoroughly mixed. Find the probability that the 2 randomly chosen blocks have 2 coloured faces each.

4. A player tosses an unbiased coin and is to score two points for every head turned up and one point for every tail turned up. If P_n denotes the probability that his score is exactly n points, prove that

$$P_n - P_{n-1} = \frac{1}{2}(P_{n-2} - P_{n-1}),\ n \geq 3$$

Also compute P_1 and P_2 and hence deduce the probability that he scores exactly 4.

5. The probabilities that three men hit a target are, respectively, 0.3, 0.5 and 0.4. Each fires once at the target. (As usual, assume that the three events that each hits the target are independent)

(a) Find the probability that they all : (i) hit the target; (ii) miss the target

(b) Find the probability that the target is hit : (i) atleast once, (ii) exactly once.

(c) If only one hits the target, what is the probability that is was the first man ?

6. Let A & B be two events defined on a sample space. Given P(A) = 0.4 ; P(B) = 0.80 and $P\left(\bar{A}/\bar{B}\right) = 0.10$.

Then find ; (i) $P\left(\bar{A} \cup B\right)$ & (ii) $P\left[\left(\bar{A} \cap B\right) \cup \left(A \cap \bar{B}\right)\right]$.

7. Three shots are fired at a target in succession. The probabilities of a hit in the first shot is $\frac{1}{2}$, in the second $\frac{2}{3}$ and in the third shot is $\frac{3}{4}$. In case of exactly one hit, the probability of destroying the target is $\frac{1}{3}$ and in the case of exactly two hits $\frac{7}{11}$ and in the case of three hits is 1.0. Find the probability of destroying the target in three shots.

8. A certain drug, manufactured by a Company is tested chemically for its toxic nature. Let the event "THE DRUG IS TOXIC" be denoted by H & the event "THE CHEMICAL TEST REVEALS THAT THE DRUG IS TOXIC" be denoted by S. Let P(H) = a, P(S/H) = $P(\bar{S}/\bar{H})$ = 1 – a. Then show that the probability that the drug is not toxic given that the chemical test reveals that it is toxic is free from 'a'.

9. A plane is landing. If the weather is favourable, the pilot landing the plane can see the runway. In this case the probability of a safe landing is p_1. If there is a low cloud ceiling, the pilot has to make a blind landing by instruments. The reliability (the probability of failure free functioning) of the instruments needed for a blind landing is P. If the blind landing instruments function normally, the plane makes a safe landing with the same probability p_1 as in the case of a visual landing. If the blind landing instruments fail, then the pilot may make a safe landing with probability $p_2 < p_1$. Compute the probability of a safe landing if it is known that in K percent of the cases there is a low cloud ceiling. Also find the probability that the pilot used the blind landing instruments, if the plane landed safely.

10. A is a set containing n distinct elements. A non-zero subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A non-zero subset Q of A is again chosen at random. Find the probability that P & Q have no common elements.

11. In a multiple choice question there are five alternative answers of which one or more than one is correct. A candidate will get marks on the question only if he ticks the correct answers. The candidate ticks the answers at random. If the probability of the candidate getting marks on the question is to be greater than or equal to 1/3, find the least number of chances he should be allowed.

12. The ratio of the number of trucks along a highway, on which a petrol pump is located, to the number of cars running along the same highway is 3 : 2. It is known that an average of one truck in thirty trucks and two cars in fifty cars stop at the petrol pump to be filled up with the fuel. If a vehicle stops at the petrol pump to be filled up with the fuel, find the probability that it is a car.

13. There are 6 red balls & 8 green balls in a bag. 5 balls are drawn out at random & placed in a red box ; the remaining 9 balls are put in a green box. What is the probability that the number of red balls in the green box plus the number of green balls in the red box is not a prime number ?

14. (a) Two natural numbers x & y are chosen at random. Find the probability that $x^2 + y^2$ is divisible by 10.

(b) Two numbers x & y are chosen at random from the set {1,2,3,4,....3n}. Find the probability that $x^2 - y^2$ is divisible by 3.

15. A hunter's chance of shooting an animal at a distance r is $\frac{a^2}{r^2}$ (r > a). He fires when r = 2a & if he misses he reloads & fires when r = 3a, 4a,..... If he misses at a distance 'na', the animal escapes. Find the odds against the hunter.

16. There are two lots of identical articles with different amount of standard and defective articles. There are N article in the first lot, n of which are defective and M articles in the second lot, m of which are defective . K articles are selected from the first lot and L articles from the second and a new lot results. Find the probability that an article selected at random from the new lot is defective.

17. A, B are two inaccurate arithmeticians whose chances of solving a given question correctly are (1/8) and (1/12) respectively. They solve a problem and obtained the same result. If it is 1000 to 1 against their making the same mistake, find the chance that the result is correct.

18. Eight players P_1 , P_2 , P_3 ,..........P_8 play a knockout tournament. It is known that whenever the players P_i and P_j play , the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the players P_4 reaches the final ? **[JEE 99]**

BRAIN STORMING SUBJECTIVE EXERCISE **ANSWER KEY** EXERCISE-04(B)

1. $\frac{31}{300}$ **2.** (a) $\frac{125}{16^3}$ (b) $\frac{63}{64}$ **3.** $\frac{^{24}C_2}{^{64}C_2}$ or $\frac{23}{168}$ **4.** $P_1 = 1/2$, $P_2 = 3/4$

5. (a) 6%, 21% ; (b) 79%, 44% ; (c) 9/44 ≈ 20.45% **6.** (i) 0.82 (ii) 0.76 **7.** $\frac{5}{8}$ **8.** $P(\bar{H}/S) = 1/2$

9. $P(E) = \left(1-\frac{K}{100}\right)p_1 + \frac{K}{100}[P\,p_1 + (1-P)\,p_2]$; $P(H_2/A) = \dfrac{\frac{K}{100}[Pp_1 + (1-P)p_2]}{\left(1-\frac{K}{100}\right)p_1 + \frac{K}{100}[Pp_1 + (1-P)p_2]}$

10. $(3^n - 2^{n+1} + 1)/(4^n - 2^{n+1} + 1)$ **11.** 11 **12.** $\frac{4}{9}$ **13.** 213/1001

14. (a) 9/50 (b) $\frac{(5n-3)}{(9n-3)}$ **15.** n+1 : n−1 **16.** $\frac{KnM + LmN}{MN(K+L)}$ **17.** $\frac{13}{14}$ **18.** 4/35

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. A problem in Mathematics is given to three students A, B, C and their respectively probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is- [AIEEE 2002]

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $\frac{2}{3}$ (4) $\frac{1}{3}$

2. If A and B are events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, $P(\overline{A}) = \frac{2}{3}$, then $P(\overline{A} \cap B)$ is- [AIEEE 2002]

(1) $\frac{5}{12}$ (2) $\frac{3}{8}$ (3) $\frac{5}{8}$ (4) $\frac{1}{4}$

3. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is- [AIEEE 2002]

(1) $\frac{8}{3}$ (2) $\frac{3}{8}$ (3) $\frac{4}{5}$ (4) $\frac{5}{4}$

4. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is- [AIEEE 2003]

(1) $\frac{4}{5}$ (2) $\frac{3}{5}$ (3) $\frac{1}{5}$ (4) $\frac{2}{5}$

5. If $\frac{(1+3p)}{3}$, $\frac{(1-p)}{4}$ and $\frac{(1-2p)}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is- [AIEEE 2003]

(1) $\frac{1}{3} \le p \le \frac{1}{2}$ (2) $\frac{1}{3} < p < \frac{1}{2}$ (3) $\frac{1}{2} \le p \le \frac{2}{3}$ (4) $\frac{1}{2} < p < \frac{2}{3}$

6. A speaks truth in 75% cases and B in 80% cases. What is the probability that they contradict each other in stating the same fact ? [AIEEE 2004]

(1) $\frac{7}{20}$ (2) $\frac{13}{20}$ (3) $\frac{3}{20}$ (4) $\frac{1}{5}$

7. A random variable X has the probability distribution :

X :	1	2	3	4	5	6	7	8
p(X) :	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and F = {X < 4}, the probability $P(E \cup F)$ is- [AIEEE 2004]

(1) 0.35 (2) 0.77 (3) 0.87 (4) 0.50

8. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is- [AIEEE 2004]

(1) $\frac{128}{256}$ (2) $\frac{219}{256}$ (3) $\frac{37}{256}$ (4) $\frac{28}{256}$

9. Let A and B be two events such that $P\left(\overline{A \cup B}\right) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then events A and B are- [AIEEE 2005]

(1) mutually exclusive and independent (2) independent but not equally likely

(3) equally likely but not independent (4) equally likely and mutually exclusive

10. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is- **[AIEEE 2005]**

(1) 7/9 (2) 8/9 (3) 1/9 (4) 2/9

11. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. the probability that the targert is hit by the second plane is :- **[AIEEE-2007]**

(1) 0.14 (2) 0.2 (3) 0.7 (4) 0.06

12. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(\frac{A}{B}) = \frac{1}{2}$ and $P(\frac{B}{A}) = \frac{2}{3}$. Then P(B) is **[AIEEE-2008]**

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{1}{2}$

13. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is **[AIEEE-2008]**

(1) $\frac{3}{5}$ (2) 0 (3) 1 (4) $\frac{2}{5}$

14. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals **[AIEEE-2009]**

(1) 5/14 (2) 1/50 (3) 1/14 (4) 1/7

15. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than **[AIEEE-2009]**

(1) $\frac{9}{\log_{10}4 - \log_{10}3}$ (2) $\frac{4}{\log_{10}4 - \log_{10}3}$ (3) $\frac{1}{\log_{10}4 - \log_{10}3}$ (4) $\frac{1}{\log_{10}4 + \log_{10}3}$

16. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have difference colours is :- **[AIEEE-2010]**

(1) $\frac{1}{3}$ (2) $\frac{2}{7}$ (3) $\frac{1}{21}$ (4) $\frac{2}{23}$

17. Four numbers are chosen at random (without replacement) from the set (1, 2, 3, ..., 20).

Statement–1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$

Statement–2 : In the four chosen numbers form an AP, then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}. **[AIEEE-2010]**

(1) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.
(2) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for statement–1.
(3) Statement–1 is true, Statement–2 is false.
(4) Statement–1 is false, Statement–2 is true.

18. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is :- **[AIEEE-2011]**

(1) $P(C\,|\,D) < P(C)$ (2) $P(C\,|\,D) = \frac{P(D)}{P(C)}$ (3) $P(C|D) = P(C)$ (4) $P(C|D) \geq P(C)$

19. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval :- [AIEEE-2011]

(1) $\left[0,\frac{1}{2}\right]$ (2) $\left(\frac{11}{12},1\right]$ (3) $\left(\frac{1}{2},\frac{3}{4}\right]$ (4) $\left(\frac{3}{4},\frac{11}{12}\right]$

20. Let A, B, C be pairwise independent events with P(C)> 0 and $P(A \cap B \cap C)=0$. Then $P(A^c \cap B^c | C)$ is equal to: [AIEEE-2011]

(1) $P(A^c) - P(B)$ (2) $P(A) - P(B^c)$ (3) $P(A^c) + P(B^c)$ (4) $P(A^c) - P(B^c)$

21. Three numbers are chosen at random without replacement from {1, 2, 3, ... , 8}. The probability that their minimum is 3, given that their maximum is 6, is : [AIEEE-2012]

(1) $\frac{2}{5}$ (2) $\frac{3}{8}$ (3) $\frac{1}{5}$ (4) $\frac{1}{4}$

22. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is : [JEE (Main)-2013]

(1) $\frac{17}{3^5}$ (2) $\frac{13}{3^5}$ (3) $\frac{11}{3^5}$ (4) $\frac{10}{3^5}$

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-5 [A]

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	1	1	4	4	1	1	2	4	2	3	1	2	3	3	3
Que.	16	17	18	19	20	21	22								
Ans	2	3	4	1	1	3	3								

EXERCISE - 05 [B] JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. A coin has probability 'p' of showing head when tossed. If is tossed 'n' times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that , $p_1 = 1$, $p_2 = 1 - p^2$ & $p_n = (1 - p)\, p_{n-1} + p\,(1 - p)\, p_{n-2}$, for all $n \geq 3$. **[JEE 2000 (Mains), 5M]**

2. (a) An urn contains 'm' white and 'n' black balls. A ball is drawn at random and is put back into the urn along with K additional balls of the same color as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white. **[JEE 2001 (Mains), 5M]**

   (b) An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6 is thrown n times and the list of n numbers showing up is noted. What is the probability that among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in the list. **[JEE 2001 (Mains), 5M]**

3. A box contains N coins, m of which are fair and the rest are biased . The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair ? **[JEE 2002 (Screening)]**

4. (a) For a student to qualify , he must pass at least two out of three exams. The probability that he will pass the 1^{st} exam is p . If he fails in one of the exams then the probability of his passing in the next exam is $\frac{p}{2}$ otherwise it remains the same. Find the probability that he will qualify.

   (b) A is targeting to B, B and C are targeting to A . Probability of hitting the target by A, B and C are $\frac{2}{3}, \frac{1}{2}, \frac{1}{3}$ respectively. If A is hit then find the probability that B hits the target and C does not.

   [JEE 2003, Mains-2 + 2 out of 60]

5. (a) If A & B are two independent events, prove that $P(A \cup B).P(A' \cap B') \leq P(C)$, where C is an event defined as exactly one of A or (and) B occurs.

   (b) A bag contains 12 red balls and 6 white balls. Six balls are drawn one by one without replacement of which atleast four balls are white. Find the probability that in the next two draws exactly one white ball is drawn. (Leave the answer in terms of ${}^{n}C_{r}$.)

   (c) Three distinct numbers are selected from first hundred natural numbers, then probability of selected numbers divisible by both 2 and 3 is - **[JEE-2004, 2 + 4 + 3]**

   (A) $\frac{4}{25}$ (B) $\frac{4}{35}$ (C) $\frac{4}{55}$ (D) $\frac{4}{1155}$

6. (a) A fair dice is thrown until 1 comes, then probability that 1 comes in even number of trials is -

   (A) 5/11 (B) 5/6 (C) 6/11 (D) 1/6

   [JEE 2005 (Screening)]

   (b) A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car.

   [JEE 2005 (Mains) 2M out of 60]

7. There are n urns each containing n + 1 balls such that the i^{th} urn contains i white balls and (n + 1 − i) red balls. Let u_i, be the event of selecting ith urn, i = 1, 2, 3,......,n and w denotes the event of getting a white ball **[JEE 2006, 15M out of 184]**

(a) If $P(u_i) \propto i$, where i = 1, 2, 3,......,n, then $\lim\limits_{n\to\infty} P(w)$ is equal to -

(A) 1 (B) 2/3 (C) 3/4 (D) 1/4

(b) If $P(u_i)$ = c where c is a constant then $P(u_n/w)$ is equal to -

(A) $\frac{2}{n+1}$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{2}$

(c) If n is even and E denotes the event of choosing even numbered urn $\left(P(u_i)=\frac{1}{n}\right)$, then the value of P(w/E) is -

(A) $\frac{n+2}{2n+1}$ (B) $\frac{n+2}{2(n+1)}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{n+1}$

8. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is - **[JEE 2007, 3M]**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$

9. Let E^c denote the complement of an event E. Let E,F,G be pairwise independent events with P(G) > 0 and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c | G)$ equals :- **[JEE 2007, 3M]**

(A) $P(E^c) + P(F^c)$ (B) $P(E^c) - P(F^c)$ (C) $P(E^c) - P(F)$ (D) $P(E) - P(F^c)$

10. Let $H_1, H_2, \ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, i = 1,2...,n. Let E be any other event with 0 < P (E) < 1.

Statement-I :

$P(H_i|E) > P(E|H_i).P(H_i)$ for i = 1,2,...,n...

because

Statement-II : $\sum\limits_{i=1}^{n} P(H_i) = 1$ **[JEE 2007, 3M]**

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

11. Consider the system of equations ax + by = 0, cx + dy = 0, where a, b, c, d ∈ {0, 1}

Statement-I : The probability that the system of equations has a unique solution is $\frac{3}{8}$.

because

Statement-II : The probability that the system of equations has a solution is 1. **[JEE 2008, 3M]**

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

12. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is - **[JEE 2008, 3M, –1M]**

(A) 2, 4, or 8 (B) 3, 6 or 9 (C) 4 or 8 (D) 5 or 10

Comprehension : (Q.N. 13 to 15)

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

13. The probability that X = 3 equals - **[JEE 2009, 4M, –1M]**

(A) $\frac{25}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{125}{216}$

14. The probability that $X \geq 3$ equals - **[JEE 2009, 4M, –1M]**

(A) $\frac{125}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{25}{216}$

15. The conditional probability that $X \geq 6$ given $X > 3$ equals - **[JEE 2009, 4M, –1M]**

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

16. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is- **[JEE 2010, 3M, –1M]**

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

17. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is -
[JEE 2010, 5M, –2M]

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Paragraph for Question 18 and 19

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

18. The probability of the drawn ball from U_2 being white is -

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

19. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is -

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$ **[JEE 2011, 3+3]**

20. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then - **[JEE 2011, 4M]**

(A) $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{5}$ (B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$ (D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

21. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and X_1, X_2, X_3 denotes respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true?
[JEE 2012, 4M]

(A) $P\left[X_1^c \mid X\right] = \frac{3}{16}$ (B) P[Exactly two engines of ship are functioning $\mid X] = \frac{7}{8}$

(C) $P[X \mid X_2] = \frac{5}{16}$ (D) $P[X \mid X_1] = \frac{7}{16}$

22. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is - **[JEE 2012, 4M]**

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

23. Let X and Y be two events such that $P(X \mid Y)=\frac{1}{2}, P(Y \mid X)=\frac{1}{3}$ and $P(X \cap Y)=\frac{1}{6}$. Which of the following is(are) correct ? **[JEE 2012, 4M]**

(A) $P(X \cup Y)=\frac{2}{3}$ (B) X and Y are independent

(C) X and Y are not independent (D) $P(X^C \cap Y)=\frac{1}{3}$

24. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is **[JEE(Advanced) 2013, 2M]**

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{3}{256}$ (D) $\frac{253}{256}$

25. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)\ p = \alpha\beta$ and $(\beta - 3\gamma)\ p = 2\beta\gamma$. All the given probabilities are assumed of lie in the interval (0,1).

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$ **[JEE-Advanced 2013, 4, (–1)]**

Paragraph for Question 26 and 27

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 4 black balls.

26. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$ (C) $\frac{65}{181}$ (D) $\frac{55}{181}$

[JEE(Advanced) 2013, 3, (–1)]

27. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$ (C) $\frac{558}{648}$ (D) $\frac{566}{648}$

[JEE(Advanced) 2013, 3, (–1)]

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-05 [B]

2. (a) $\frac{m}{m+n}$; (b) $\frac{{}^6C_3(3^n - 3.2^n + 3)}{6^n}$ **3.** $\frac{9m}{m+8N}$ **4.** (a) $2p^2 - p^3$; (b) 1/2

5. (b) $\frac{{}^{10}C_1 . {}^2C_1 . {}^6C_4 . {}^{12}C_2 + {}^{11}C_1 . {}^1C_1 . {}^6C_5 . {}^{12}C_1}{{}^{12}C_2\left[{}^6C_4 . {}^{12}C_2 + {}^6C_5 . {}^{12}C_1 + {}^6C_6 . {}^{12}C_0\right]}$ (c) D **6.** (a) A (b) $\frac{1}{7}$ **7.** (a) B (b) A (c) B

8. C **9.** C **10.** D **11.** B **12.** D **13.** A **14.** B **15.** D **16.** C **17.** C **18.** B
19. D **20.** A,D **21.** B,D **22.** A **23.** A,B **24.** A **25.** 6 **26.** D **27.** A